Online Supplement to

Uncertainty and Economic Activity: A Multi-Country Perspective

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Introduction

This supplement gives data sources and some summary statistics, and provides details of robustness analysis, country-specific results, and the derivation of impulse responses and error variance decompositions for global and country-specific shocks used in the paper.

S.1 Data Sources and Summary Statistics

Data Sources To construct a balanced panel for the largest number of countries for which we have sufficiently long time series, we first collect daily stock prices (excluding dividends) for 32 advanced and emerging economies from 1979 to 2016. We then cut the beginning of the sample in 1993, as daily equity price data are not available earlier for two large emerging economies (Brazil and China) and for Peru. Better quality quarterly GDP data for China also became available from 1993.[S1]

For equity prices we use the MSCI Index in local currency. We collected daily observations from January 1993 to December 2016. The data source for the daily equity price indices is Datastream. The countries included in the sample are the following: Argentina, Australia, Austria, Belgium, Brazil, Canada, Chile, China, Finland, France, Germany, India, Indonesia, Italy, Japan, Korea, Malaysia, Mexico, Netherlands, Norway, New Zealand, Peru, Philippines, South Africa, Singapore, Spain, Sweden, Switzerland, Thailand, Turkey, United Kingdom, and United States. The list of Bloomberg tickers is as follows: TOTMKAR, TOTMKAU, TOTMKOE, TOTMKBG, TOTMKBR, TOTMKCN, TOTMKCL, TOTMKCA, TOTMKFN, TOTMKFR, TOTMKBD, TOTMKIN, TOTMKID, TOTMKIT, TOTMKJP, TOTMKKO, TOTMKMY, TOTMKMX, TOTMKNL, TOTMKNZ, TOTMKNW, TOTMKPE, TOTMKPH, TOTMKSG, TOTMKSA, TOTMKES, TOTMKSD, TOTMKSW, TOTMKTH, TOTMKTK, TOTMKUK, TOTMKU.

Real GDP data come from the latest update of the GVAR data set. The data set is balanced and good quality quarterly data are available for all countries in our sample from 1993:Q1 to 2016:Q4. For more details see: https://sites.google.com/site/gvarmodelling/.

Cross-country Correlations The differential pattern of cross-country correlations of the growth and volatility innovations is crucial for our identification strategy. Here we consider the properties of the observed time series as displayed in Figure 1 in the paper. In order to gauge the extent to which volatility and growth series co-move across countries, we use two techniques: standard principal component analysis and pair-wise correlation analysis across countries.

In a panel of countries indexed by $i = 1, 2, ..., N$, the average pair-wise correlation of country i in the panel ($\bar{\rho}_i$) measures the average degree of comovement of country i with all other countries j (i.e.

[S1] Note that some steps of the empirical analysis can be easily implemented with the unbalanced panel from 1979. This is the case, for example, for the estimates of factor innovations ($\hat{\zeta}_t$ and $\hat{\xi}_t$), which we report in Section S.2 below.

for all $j \neq i$). The average pair-wise correlation across all countries, denoted by $\bar{\rho}_N$, is defined as the cross-country average of $\bar{\rho}_i$ over $i = 1, 2, ..., N$. This statistic relates to the degree of pervasiveness of the factors, as measured by the factor loadings. To see this, consider equation (2) of our model, $\Delta y_{it} = \gamma_i f_t + \varepsilon_{it}$, where $Var(f_t) = 1$, and $Var(\varepsilon_{it}) = \sigma_{ii}$.[S2] The average pair-wise correlation across all countries is given by:

$$\bar{\rho}_N = \frac{2}{N(N-1)} \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} \rho_{ij} = \frac{1}{N(N-1)} \left(\sum_{i=1}^{N} \sum_{j=1}^{N} \rho_{ij} - N \right), \tag{S1}$$

where

$$\rho_{ij} = \begin{cases} \frac{\tilde{\gamma}_i}{\sqrt{1+\tilde{\gamma}_i^2}} \frac{\tilde{\gamma}_j}{\sqrt{1+\tilde{\gamma}_j^2}} & \text{if } i \neq j \\ 1 & \text{if } i = j \end{cases}$$

and $\tilde{\gamma}_i = \gamma_i / \sqrt{\sigma_{ii}}$. Hence

$$\bar{\rho}_N = O\left(\bar{\gamma}_N^2\right), \tag{S2}$$

where $\bar{\gamma}_N = N^{-1} \sum_{i=1}^{N} \tilde{\gamma}_i$ measures the degree of pervasiveness of the factor.

The attraction of the average pair-wise correlation, $\bar{\rho}_N$, lies in the fact that it applies to multi-factor processes, and unlike factor analysis does not require the factors to be strong. In fact, the average pair-wise correlation, $\bar{\rho}_N$, tends to be a strictly positive number if Δy_{it} contains at least one strong factor, otherwise it tends to zero as $N \to \infty$. Therefore, non-zero estimates of $\bar{\rho}_N$ are suggestive of strong cross-sectional dependence.[S3] For completeness, and to show that our analysis is robust to using an alternative methodology, in what follows, we also use standard principal component analysis. (See also Chapter 29 in Pesaran (2015) for more details).

The average pair-wise correlation across all countries for the realized volatility series in Figure 1 is 0.56. In contrast, the average pair-wise correlation across all countries for the growth series at 0.27 is much smaller. Principal component analysis yields similar results. The first principal component in our panel of realized volatility series explains 65 percent of the total variation in the log-level of volatility, whilst the first principal component of the growth series accounts for only around 30 percent of total cross-country variations in these series. Thus, both in the case of the pair-wise correlation and principal component analysis, the results point to a much higher degree of cross-country comovements for the volatility series than for the growth series. As we will see, these differences are even more pronounced in the case of the estimated shocks obtained using equations (43) and (44).

Summary Statistics Table S.1 reports the summary statistics for the realized volatility series for each country in our sample. These results support the use of the log-level of realized volatilities as stationary series in our empirical analysis. Tables S.2 and S.3 give similar summary statistics for log of real GDP and its growth rate, and justifies using the latter as a stationary variable along with the log of realized volatility.

[S2] Under our assumptions $Var_t(\varepsilon_{i,t+1}) = \theta_t^2 \sigma_{ii}$, which gives $Var(\varepsilon_{i,t+1}) = \sigma_{ii}$.

[S3] Formal tests of cross-sectional dependence based on estimates of $\bar{\rho}_N$ are discussed in Pesaran (2015) and reported, for our panel of countries, in the next section.

Table S.1 Summary Statistics for Country-specific Realized Volatility (Log-level)

	ARG	AUS	AUT	BEL	BRA	CAN	CHL	CHN	FIN	FRA	DEU
Obs. in quarters	96	96	96	96	96	96	96	96	96	96	96
Mean	-2.12	-2.74	-2.70	-2.68	-2.20	-2.78	-2.85	-2.09	-2.17	-2.48	-2.53
Max	-1.23	-1.42	-1.17	-1.41	-1.12	-1.16	-1.77	-0.60	-1.09	-1.27	-1.23
Min	-2.86	-3.52	-3.38	-3.55	-2.84	-3.53	-3.56	-3.01	-2.87	-3.22	-3.44
St. Dev.	0.36	0.36	0.43	0.45	0.42	0.44	0.38	0.48	0.46	0.38	0.41
Auto Corr.	0.37	0.64	0.66	0.70	0.64	0.70	0.46	0.66	0.77	0.61	0.64
ADF	-3.12†	-2.37	-2.17	-2.69‡	-3.55*	-2.64‡	-3.19†	-2.69‡	-2.31	-2.91‡	-2.79‡
	IND	IDN	ITA	JPN	KOR	MYS	MEX	NLD	NZL	NOR	PER
Obs. in quarters	96	96	96	96	96	96	96	96	96	96	96
Mean	-2.31	-2.25	-2.36	-2.40	-2.21	-2.76	-2.52	-2.56	-2.98	-2.42	-2.72
Max	-1.27	-1.07	-1.27	-1.13	-1.03	-0.82	-1.46	-1.21	-1.68	-0.93	-1.39
Min	-3.06	-3.19	-3.32	-3.17	-3.06	-3.87	-3.40	-3.33	-3.75	-3.09	-3.79
St. Dev.	0.40	0.43	0.40	0.34	0.48	0.59	0.43	0.46	0.38	0.39	0.49
Auto Corr.	0.58	0.60	0.58	0.32	0.79	0.72	0.66	0.69	0.67	0.61	0.67
ADF	-2.73‡	-2.46	-2.52	-3.6*	-1.50	-2.24	-2.36	-2.79‡	-3.09†	-2.81‡	-2.56
	PHL	SGP	ZAF	ESP	SWE	CHE	THA	TUR	GBR	USA	
Obs. in quarters	96	96	96	96	96	96	96	96	96	96	
Mean	-2.46	-2.65	-2.53	-2.42	-2.35	-2.67	-2.20	-1.83	-2.67	-2.64	
Max	-1.54	-1.44	-1.45	-1.35	-1.21	-1.41	-1.20	-0.90	-1.28	-1.10	
Min	-3.22	-3.47	-3.23	-3.18	-3.02	-3.39	-3.12	-2.81	-3.35	-3.40	
St. Dev.	0.37	0.45	0.37	0.39	0.40	0.41	0.42	0.45	0.42	0.44	
Auto Corr.	0.46	0.68	0.53	0.58	0.66	0.54	0.64	0.73	0.66	0.69	
ADF	-2.97†	-2.06	-3.28†	-2.56	-2.59‡	-3.23†	-2.25	-1.43	-2.56	-2.27	

Note. Summary statistics of the log-level of volatility (v_{it}). *ADF* is the Augmented Dickey-Fuller t-statistic computed with 4 lags and a constant, where $*$, †, and ‡ denote associated p-values at 1-percent, 5-percent, and 10-percent. Sample period 1993:Q1-2016:Q4.

Table S.2 Summary Statistics for Country-specific Real GDP (Log-Level)

	ARG	AUS	AUT	BEL	BRA	CAN	CHL	CHN	FIN	FRA	DEU
Obs. in quarters	96	96	96	96	96	96	96	96	96	96	96
Mean	4.78	4.73	4.66	4.65	4.75	4.67	4.79	5.14	4.65	4.64	4.64
Max	5.20	5.08	4.84	4.82	5.05	4.92	5.24	6.26	4.84	4.77	4.79
Min	4.42	4.30	4.40	4.40	4.39	4.32	4.23	3.99	4.29	4.43	4.47
St. Dev.	0.27	0.23	0.13	0.12	0.20	0.18	0.30	0.72	0.16	0.11	0.09
Auto Corr.	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
ADF	-0.30	-3.57*	-1.86	-1.76	-1.83	-2.12	-1.62	-0.24	-2.20	-2.00	-0.72
	IND	IDN	ITA	JPN	KOR	MYS	MEX	NLD	NZL	NOR	PER
Obs. in quarters	96	96	96	96	96	96	96	96	96	96	96
Mean	4.91	4.88	4.60	4.65	4.77	4.79	4.67	4.62	4.98	4.59	4.86
Max	5.73	5.46	4.69	4.75	5.19	5.35	4.96	4.78	5.26	4.80	5.44
Min	4.08	4.37	4.46	4.55	4.19	4.15	4.31	4.36	4.63	4.23	4.24
St. Dev.	0.48	0.31	0.06	0.06	0.28	0.32	0.19	0.12	0.17	0.15	0.35
Auto Corr.	1.00	1.00	0.99	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00
ADF	-0.37	0.64	-2.44	-0.78	-1.63	-0.59	-0.31	-1.97	-0.98	-2.8‡	0.21
	PHL	SGP	ZAF	ESP	SWE	CHE	THA	TUR	GBR	USA	
Obs. in quarters	96	96	96	96	96	96	96	96	96	96	
Mean	4.84	4.78	4.77	4.66	4.68	4.67	4.82	4.78	4.66	4.66	
Max	5.45	5.31	5.07	4.84	4.97	4.85	5.18	5.28	4.86	4.89	
Min	4.33	4.06	4.39	4.37	4.36	4.48	4.37	4.27	4.37	4.33	
St. Dev.	0.32	0.36	0.22	0.15	0.17	0.12	0.23	0.29	0.14	0.16	
Auto Corr.	1.00	1.00	1.00	1.00	1.00	1.00	0.99	0.99	1.00	1.00	
ADF	1.96	-1.37	-1.36	-1.65	-0.54	-0.61	-0.68	-0.59	-1.77	-1.65	

Note. Summary statistics for the log-level of real GDP (y_{it}). *ADF* is the Augmented Dickey-Fuller t-statistic computed with 4 lags and a constant, where $*$, †, and ‡ denote associated p-values at 1-percent, 5-percent, and 10-percent. Sample period 1993:Q1-2016:Q4.

Table S.3 Summary Statistics for Country-specific Real GDP (Log-Difference)

	ARG	AUS	AUT	BEL	BRA	CAN	CHL	CHN	FIN	FRA	DEU
Obs. in quarters	95	95	95	95	95	95	95	95	95	95	95
Mean	0.78	0.81	0.46	0.43	0.60	0.63	1.06	2.40	0.55	0.36	0.34
Max	4.04	2.46	2.48	2.23	4.83	1.64	5.91	5.91	4.41	1.56	2.19
Min	-6.35	-0.99	-2.61	-2.12	-5.19	-2.27	-3.38	-1.33	-6.01	-1.70	-4.16
St. Dev.	1.92	0.56	0.92	0.69	1.44	0.61	1.28	1.24	1.32	0.50	0.74
Auto Corr.	0.59	-0.04	0.00	0.24	0.25	0.54	0.12	0.08	0.12	0.47	0.33
ADF	-3.49*	-3.75*	-3.68*	-4.64*	-3.63*	-3.78*	-3.18†	-2.45	-3.88*	-3.19†	-4.65*
	IND	IDN	ITA	JPN	KOR	MYS	MEX	NLD	NZL	NOR	PER
Obs. in quarters	95	95	95	95	95	95	95	95	95	95	95
Mean	1.73	1.14	0.18	0.18	1.05	1.26	0.63	0.44	0.67	0.60	1.27
Max	5.41	5.11	1.79	2.53	3.81	4.65	3.77	1.70	2.54	4.48	4.44
Min	-1.43	-8.17	-3.70	-4.09	-8.94	-7.10	-6.07	-2.10	-1.67	-1.74	-2.52
St. Dev.	1.28	1.76	0.69	1.00	1.48	1.59	1.43	0.65	0.71	1.15	1.36
Auto Corr.	-0.06	0.34	0.42	0.29	0.27	0.33	0.33	0.58	0.24	-0.20	0.28
ADF	-4.66*	-3.47*	-3.3†	-5.08*	-4.74*	-5.34*	-4.1*	-3.04†	-3.94*	-3.46*	-4.2*
	PHL	SGP	ZAF	ESP	SWE	CHE	THA	TUR	GBR	USA	
Obs. in quarters	95	95	95	95	95	95	95	95	95	95	
Mean	1.17	1.32	0.71	0.50	0.64	0.39	0.86	0.99	0.52	0.59	
Max	3.21	6.77	1.86	2.49	2.94	1.98	10.79	6.57	1.41	1.81	
Min	-2.44	-3.77	-1.63	-1.57	-3.71	-3.50	-11.97	-11.93	-2.11	-2.18	
St. Dev.	0.87	2.03	0.60	0.60	1.12	0.78	2.47	2.91	0.58	0.59	
Auto Corr.	0.11	0.23	0.60	0.80	-0.01	0.16	-0.02	-0.01	0.69	0.41	
ADF	-4.13*	-5.28*	-2.7‡	-2.47	-4.9*	-4.15*	-4.11*	-5.21*	-4*	-3.34†	

Note. Summary statistics for the log-difference of real GDP (Δy_{it}). *ADF* is the Augmented Dickey-Fuller t-statistic computed with 4 lags and a constant, where $*$, †, and ‡ denote associated p-values at 1-percent, 5-percent, and 10-percent. Sample period 1993:Q1-2016:Q4.

S.2 Robustness Analysis

We report here the results from a few exercises showing robustness of our results.

S.2.1 Robustness to Choice of Countries (Granularity Assumptions)

This section compares the results from four robustness exercises with respect to the choice of the countries in our sample with the estimates reported in the paper that are based on all countries. In particular, we consider the following cases: (1) exclude the United States from the sample; (2) exclude China from the sample; (3) exclude the United States and China from the sample; and (4) treat the United States as the global factor, namely substitute $\hat{\zeta}_t$ and $\hat{\xi}_t$ with $\Delta y_{US,t}$ and $v_{US,t}$, respectively.

Table S.4 shows that in cases (1), (2), and (3)—i.e. when we exclude the United States, China, or both—the cross-sectional dependence of the country-specific innovations is very similar to the baseline. So, our common factors cannot be driven by shocks to these large economies. This is not true for case (4), i.e. when we treat the US economy as the common factor. In this case, Table S.4 shows that the country-specific GDP growth and volatility innovations display a significant degree of cross-sectional dependence even after conditioning on US GDP growth and US (log) volatility. Consistently with that, the CD test rejects the null of zero average pair-wise correlation of the innovations. In other words, when replacing the common factors $\hat{\zeta}_t$ and $\hat{\xi}_t$ with the US GDP growth and US volatility, we can control for some, but not all, the cross-country correlation of the GDP growth and volatility series. Table S.5 reports similar evidence based on 'long-run' (i.e. 12 quarters ahead) forecast error variance decompositions (FEVD). The Table shows that the FEVDs in cases (1), (2), and (3) are very similar to our baseline, while this is not true for case (4).

Table S.4 Cross-Sectional Dependence of the Innovations

	Pairwise Correlation			Exponent of cross-sectional dependence		
	$\hat{\varepsilon}_i$	$\hat{u}_i$	$\hat{\eta}_i$	$\hat{\varepsilon}_i$	$\hat{u}_i$	$\hat{\eta}_i$
Baseline (All countries)	-0.01	0.52	-0.02	0.56 [0.62,0.67]	0.96 [1.00,1.04]	0.58 [0.64,0.70]
Excluding US	-0.02	0.52	-0.02	0.54 [0.60,0.65]	0.96 [1.00,1.04]	0.52 [0.59,0.66]
Excluding China	-0.01	0.55	-0.03	0.57 [0.62,0.68]	0.96 [1.00,1.04]	0.61 [0.68,0.74]
Excluding US & China	-0.01	0.54	-0.03	0.56 [0.61,0.66]	0.96 [1.00,1.04]	0.57 [0.63,0.69]
US as global factor	0.15	0.49	0.27	0.91 [0.99,1.06]	0.96 [0.99,1.03]	0.96 [0.99,1.03]

Note. Pair-wise correlations and exponent of cross-sectional dependence ($\hat{\alpha}$) as in Bailey et al. (2016), together with the associated 90-percent confidence interval in square brackets. Sample period 1993:Q1-2016:Q4.

Table S.5 Forecast Error Variance Decomposition (Long-run)

	FEVD of GDP growth					
	$\hat{\xi}$	$\hat{\eta}_i$	$\sum \hat{\eta}_j$	$\hat{\zeta}$	$\hat{\varepsilon}_i$	$\sum \hat{\varepsilon}_j$
Baseline	7.56	1.94	0.14	24.88	64.62	0.86
Excluding US	7.77	1.98	0.15	24.75	64.50	0.83
Excluding China	7.46	2.03	0.14	25.57	63.86	0.92
Excluding US & China	7.66	2.08	0.15	25.42	63.79	0.89
US as global factor	5.69	2.86	0.22	6.45	83.90	0.87
	FEVD of Volatility					
	$\hat{\xi}$	$\hat{\eta}_i$	$\sum \hat{\eta}_j$	$\hat{\zeta}$	$\hat{\varepsilon}_i$	$\sum \hat{\varepsilon}_j$
Baseline	53.25	41.91	0.13	3.92	0.63	0.16
Excluding US	52.97	42.43	0.14	3.69	0.61	0.17
Excluding China	54.52	40.30	0.12	4.32	0.58	0.17
Excluding US & China	54.29	40.82	0.13	4.03	0.56	0.17
US as global factor	32.75	57.84	0.19	8.47	0.59	0.17

Note. Average across countries with GDP-PPP weights at horizon $h = 12$ quarters. $\hat{\xi}$ is common financial shock; $\hat{\eta}_i$ is country i's volatility shock; $\sum \hat{\eta}_j$ is the sum of the contribution of the volatility shocks in country j, for all $j \neq i$; $\hat{\zeta}$ is common growth shock; $\hat{\varepsilon}_i$ is country i's GDP growth shock; $\sum \hat{\varepsilon}_j$ is the sum of the contributions of the GDP growth shocks in country j, for all $j \neq i$. Sample period: 1993:Q1-2016:Q4.

S.2.2 Robustness to the Choice of Sample Periods

We report here results from a longer unbalanced sample period as well as when we exclude the global financial crisis period from the sample.

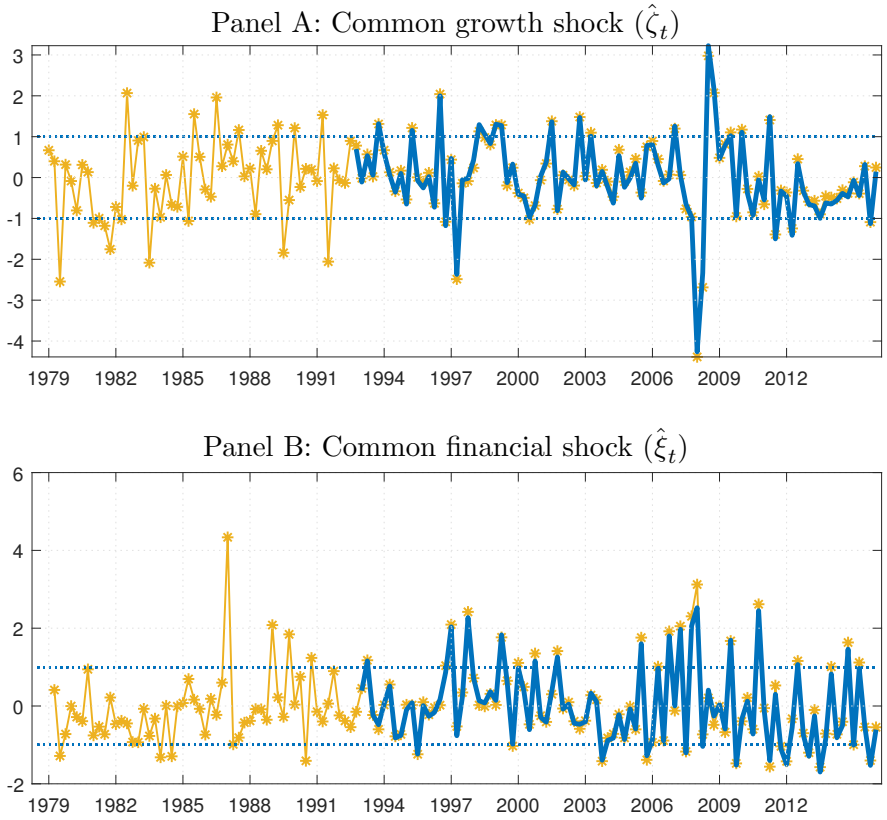
Longer-run Unbalanced Panel Estimates of the Common Shocks. In this section we consider a longer sample period starting from 1979. While for a few emerging economies quarterly GDP data is not available from this starting date, it is possible to interpolate annual series to obtain a balanced sample of GDP growth series at quarterly frequency for all countries considered in our study. For more details see: https://sites.google.com/site/gvarmodelling/. We then collected daily equity prices from January 1979 to December 2016. Note that, over this sample, it is possible to obtain a balanced panel only for 16 economies.

Estimates of the global shocks, $\hat{\zeta}_t$ and $\hat{\xi}_t$, recovered from the OLS estimation of (41) and (42) are reported in Figure S.1 when estimated using the unbalanced panel from 1979 (thin lines with asterisks), and when we use the balanced panel from 1993 (thick solid lines), so as to better illustrate their time profiles. The figure also reports one-standard deviation bands for the shocks. Note that the shocks are standardized and have zero means and unit in-sample variances. They are also serially uncorrelated and orthogonal to each other by construction. Interestingly, the Jarque-Bera test strongly rejects normality in the case of the growth shocks, with strong evidence of left skewness and kurtosis, and only marginally rejects in the case of the financial shock with mild evidence of right skewness. The figure shows that the largest negative realization of the real common shock was after the second oil shock in 1979, and during the fourth quarter of 2008 after the Lehman Brother's collapse, consistent with prevailing narratives on the characterization of world recessions. Figure S.1 illustrates that the largest realizations of the common financial shock, $\hat{\xi}_t$, coincide with the 1987 stock market crash and the 2008 Lehman Brother's collapse.

Excluding the global financial crisis. The results are robust to dropping the period of the

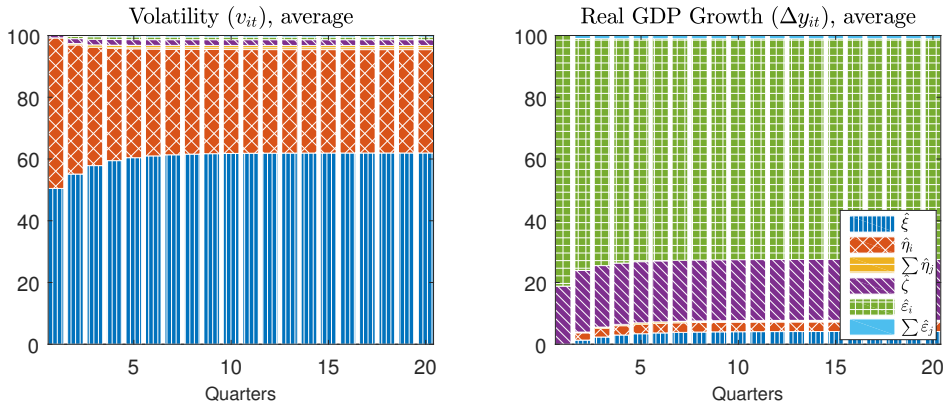
global financial crisis from our sample. For example, we report in Figures S.2 and S.3 the FEVDs and IRFs that we obtained when re-estimating the model using the sample from 1993 to 2006.

Figure S.1 Estimated Common Growth and Financial Shocks



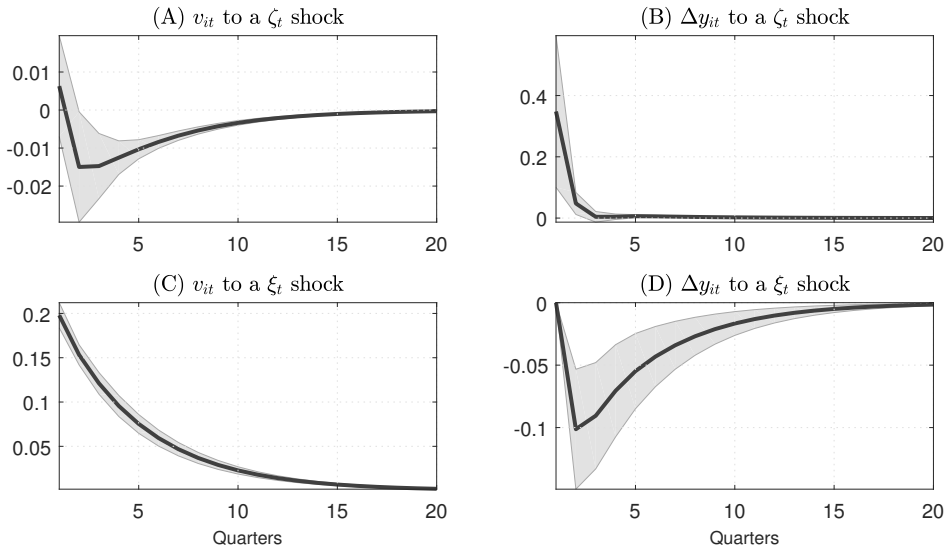
Note. The common shocks $\hat{\zeta}_t$ and $\hat{\xi}_t$ are computed using (41) and (42), with one lag of $\mathbf{z}_{it}$, using an unbalanced sample 1979:Q2-2016:Q4 (thin lines with asterisks) and the shorter balanced sample 1993:Q1-2016:Q4 (thick solid lines). The shocks are standardized and the dotted lines are the one-standard deviation bands around the zero mean.

Figure S.2 FEVD - Sample period: 1993:Q1-2006:Q4



Note. Diagonal covariance matrix. Average across countries with GDP-PPP weights. $\hat{\xi}$ is common financial shock (blue area with vertical lines); $\hat{\eta}_i$ is country-specific volatility shock (red area with crosses); $\sum \hat{\eta}_j$ is the sum of the contribution of the volatility shocks in the remaining countries (yellow area with horizontal lines); $\hat{\zeta}$ is common growth shock (purple area with diagonal lines); $\hat{\varepsilon}_i$ is country-specific GDP growth shock (green areas with squares); $\sum \hat{\varepsilon}_j$ is the sum of the contributions of the GDP growth shocks in the remaining countries (light blue areas with no pattern). The vertical axis is in percent, the horizontal axis is in quarters. Sample period: 1993:Q1-2006:Q2.

Figure S.3 Average Country Volatility and Growth Responses to Real and Financial Factor Shocks (In Percent) - Sample period: 1993:Q1-2006:Q4



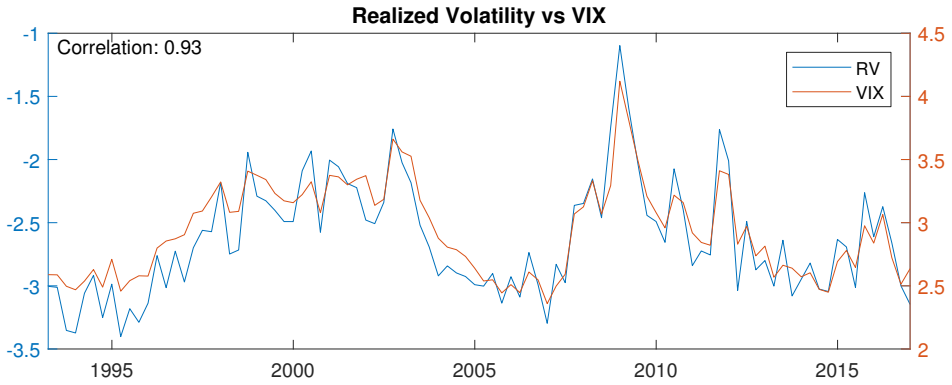
Note. Average impulse responses to one-standard deviation real and financial shocks, $\hat{\zeta}_t$ and $\hat{\xi}_t$. The solid lines are the PPP-GDP weighted averages of the country-specific responses. The shaded areas are the two standard deviations confidence intervals. See equations (S12) and (S13) for the derivations and Figure S.11 for the country-specific responses. The horizontal axis is in quarters. Sample period: 1993:Q1-2006:Q2.

S.2.3 Robustness to Choice of Uncertainty Measures: Realized *versus* Implied volatility

At quarterly frequency, the realized volatility of US daily equity returns behaves very similarly to the VIX Index. For example, during the period over which they overlap, our realized volatility measure for the U.S. and the VIX Index co-move very closely, with a correlation that exceeds 0.9. See Figure S.4. In addition, to check more formally the robustness of our results, we re-estimated our model using the VIX Index as a measure of volatility for the U.S. (instead of our realized volatility measure) and obtain virtually identical results.

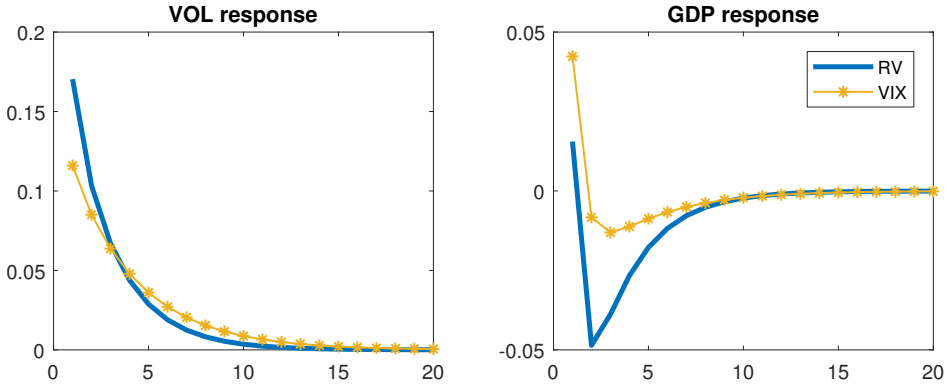
Figure S.5 compares our baseline IRFs of US volatility and US GDP growth to a US country-specific volatility shock (solid blue line) with those obtained from a specification where we used the VIX Index as a measure of US volatility instead of our realized volatility measure (yellow line with asterisks). The comparison shows that, in the robustness exercise, the correlation between US GDP and volatility residuals is even more positive than in our baseline scenario; thus reinforcing our main result.

Figure S.4 United States: VIX Index *versus* RV



Note. Blue line is the (log) realized volatility of equity prices for the United States, as in our baseline model (RV). The red line is the (log) VIX Index (average across days within the quarter). Sample period: 1993:Q1-2016:Q4.

Figure S.5 US Response to US Volatility Shock



Note. US impulse responses to a one-standard deviation shock to US volatility, $\hat{\eta}_{US,t}$. The blue lines are our baseline; the yellow lines with asterisks are obtained from a specification where we used the VIX Index as a measure of US volatility instead of our realized volatility measure. The horizontal axis is in quarters. Sample period: 1993:Q1-2016:Q4.

S.2.4 Robustness to Alternative Identification Assumptions for Country-Specific Shocks

Consider the correlation between volatility and growth innovations within each country. We saw in Figure 2 that, once we condition on the global shocks ($\hat{\zeta}_t$ and $\hat{\xi}_t$), the contemporaneous within-country correlation between $\hat{\eta}_{it}$ and $\hat{\varepsilon}_{it}$ is very small and not statistically significant in most countries. In Figure 8 we also showed that conditional on both $\hat{\zeta}_t$ and $\hat{\xi}_t$, the country-specific shocks $\hat{\varepsilon}_{it}$ and $\hat{\eta}_{it}$ are weakly correlated across countries, with average pair-wise correlations below 0.05. Weak cross-sectional dependence means that, as N grows, the overall average pair-wise correlation must tend to zero; while some pairs of correlations can be different from zero, not all pairs can be so. In practice, this means that most correlation pairs will be very small and the covariance matrix, $\mathbf{\Sigma}_{(\varepsilon,\eta)}$, in the 64 shocks $\hat{\varepsilon}_{it}$ and $\hat{\eta}_{it}$, for $i = 1, 2, ..., N$, must be sparse.

Indeed, when we apply the threshold estimation procedure of Bailey, Pesaran, and Smith (2019) to the whole set of distinct off-diagonal elements of $\mathbf{\Sigma}_{(\varepsilon,\eta)}$ we find that only 57 out of 2016 off-diagonal elements are statistically different from zero. Table S.6 shows that, of these 57, about half are positively correlated and the other half are negatively correlated, with an average value that is close to zero. Most notably, there is no surviving within-country contemporaneous correlation between volatility and growth. There are also very few significant GDP-GDP correlation pairs (i.e., $\hat{\varepsilon}_{it}$ with $\hat{\varepsilon}_{jt}$), with no obvious regional pattern of comovements. There are a few significant pairs of volatility-volatility correlations (i.e. $\hat{\eta}_{it}$ with $\hat{\eta}_{jt}$), but involving only a handful of countries, with no evidence of a dominant role for the United States. Finally, there are only two significant GDP-volatility correlation pairs (i.e. $\hat{\varepsilon}_{jt}$ with $\hat{\eta}_{it}$), again revealing no specific patterns.

Note that even a block diagonal reduced form covariance matrix (where all cross-country innovations correlations are zero), would not imply that innovations $\hat{\eta}_{it}$ and $\hat{\varepsilon}_{it}$ can be interpreted as 'structural' country-specific volatility and growth shocks. As is well known, there always exists an orthonormal transformation of η_{it} and ε_{it} that leads to the same forecast error variance decomposition. It is therefore important to complement this evidence with some explicit assumption about the 64×64 matrix of correlations.

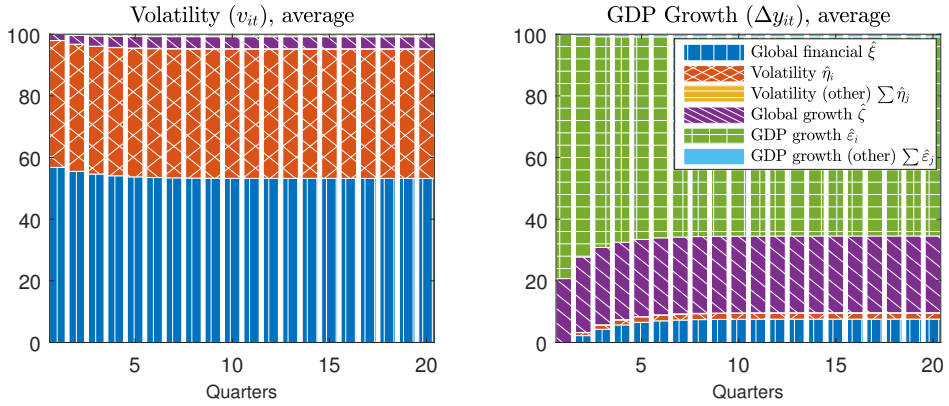
In our baseline estimates of the FEVDs we assume a block-diagonal covariance matrix for the residuals of the multi-country model (43)-(44), which amounts to assuming that (within each country) volatility shocks affect output growth contemporaneously (but not *vice versa*).

To check the robustness of our results we re-estimate the FEVDs with two alternative sets of assumptions on the covariance matrix of country-specific innovations. First, we assume that the only source of interdependence among all growth and volatility series are the global real and financial shocks $\hat{\zeta}_t$ and $\hat{\xi}_t$ and that country-specific volatility and growth shocks have no contemporaneous impact on growth or volatility series within and across countries. In other words, we assume that the reduced form innovations are also structural. Then, we also consider the case in which we refrain from interpreting these innovations structurally.[S4]

[S4]The country-specific shocks can also be identified exploiting Bayesian priors as carried out in a related study by Chudik, Mohaddes and Pesaran (2019).

Diagonal Covariance Matrix and Orthogonal Decomposition. We assume that all elements of the variance covariance matrix of the country-specific shocks are truly zero after conditioning on the common shocks across countries. The results for this specification are given in Figure S.6 and can be seen to be virtually identical to the estimates obtained for the diagonal error covariance matrix reported in Figures 4 and 6 in the paper. This is perhaps not surprising given that the correlations between the country-specific innovations, once the effects of the common shocks are removed, are very small as in Figure 2 in the paper.

Figure S.6 Forecast Error Variance Decomposition of Country-specific Shocks - Diagonal Error Covariance Matrix (In Percent)



Note. Diagonal covariance matrix. Average across countries with GDP-PPP weights. $\hat{\xi}$ is common financial shock (blue area with vertical lines); $\hat{\eta}_i$ is country-specific volatility shock (red area with crosses); $\sum \hat{\eta}_j$ is the sum of the contribution of the volatility shocks in the remaining countries (yellow area with horizontal lines); $\hat{\zeta}$ is common growth shock (purple area with diagonal lines); $\hat{\varepsilon}_i$ is country-specific GDP growth shock (green areas with squares); $\sum \hat{\varepsilon}_j$ is the sum of the contributions of the GDP growth shocks in the remaining countries (light blue areas with no pattern). The horizontal axis is in quarters. Sample period: 1993:Q1-2016:Q4.

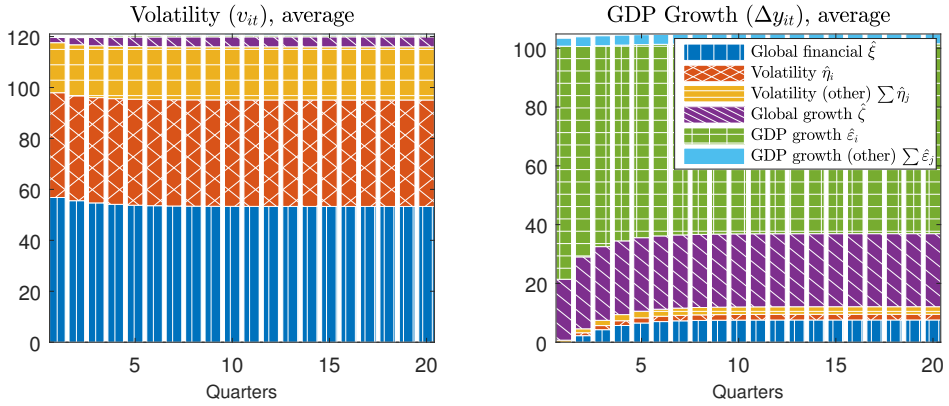
Thresholding the Country-specific Error Covariance Matrix and Generalized Error Variance Decomposition. Here we allow for a fully estimated (64×64) correlation matrix or country-specific errors, both within and across countries, and compute the Generalized Forecast Error Variance Decompositions (GFEVDs). However, given the large size of this matrix, we regularize it by computing a threshold estimator following Bailey, Pesaran, and Smith (2019), who developed a procedure based on results from the multiple testing literature. Specifically, we first test for the statistical significance of each of the 2016 distinct off-diagonal elements of the (64×64) matrix. We then set to zero all those elements that are not statistically significant, using suitably adjusted critical values to allow for the large number of tests that are being carried out. We then finally compute the GVEDs by using the regularized estimates as set out in Section S.5 below.

The estimated generalized forecast error variance decompositions (GFEVDs), reported in Figure S.7, are consistent with those obtained assuming a diagonal or block-diagonal error covariance matrix.[S5] Relative to the results with diagonal or block-diagonal covariance matrix in Figures S.6 and 4 and 6, the contribution of foreign country-specific volatility (growth) shocks, $\sum \hat{\eta}_j$ ($\sum \hat{\varepsilon}_j$), to domestic volatility (growth) is now larger, but the spillover effects of foreign volatility shocks to growth (and foreign growth shocks to volatility) remain negligible. Moreover, global financial shocks and domestic country-specific volatility shocks continue to explain the bulk of the forecast error variance of volatility. Similarly, global growth shocks and the country-specific growth shocks remain the main drivers of the forecast error variance of growth.

[S5] Notice here that the GFEVDs need not sum to 100 as the underlying shocks are not orthogonal.

We interpret the above results as strong evidence of robustness of our conclusions reached by assuming a diagonal or block-diagonal error covariance matrix. In particular, it remains the case that common or country-specific output growth shocks have a small quantitative importance for volatility, and home and foreign country-specific volatility shocks have little or no quantitative consequence for output growth.

Figure S.7 Generalized Forecast Error Variance Decomposition of Country-specific Shocks - Estimation of Regularized Full Error Covariance Matrix (In Percent)



Note. Threshold estimator of the population covariance matrix. Average across countries with GDP-PPP weights. $\hat{\xi}$ is common financial shock (blue area with vertical lines); $\hat{\eta}_i$ is country-specific volatility shock (red area with crosses); $\sum \hat{\eta}_j$ is the sum of the contribution of the volatility shocks in the remaining countries (yellow area with horizontal lines); $\hat{\zeta}$ is common growth shock (purple area with diagonal lines); $\hat{\varepsilon}_i$ is country-specific GDP growth shock (green areas with squares); $\sum \hat{\varepsilon}_j$ is the sum of the contributions of the GDP growth shocks in the remaining countries (light blue areas with no pattern). The horizontal axis is in quarters. Sample period: 1993:Q1-2016:Q4.

S.2.5 Robustness to Weighting Scheme

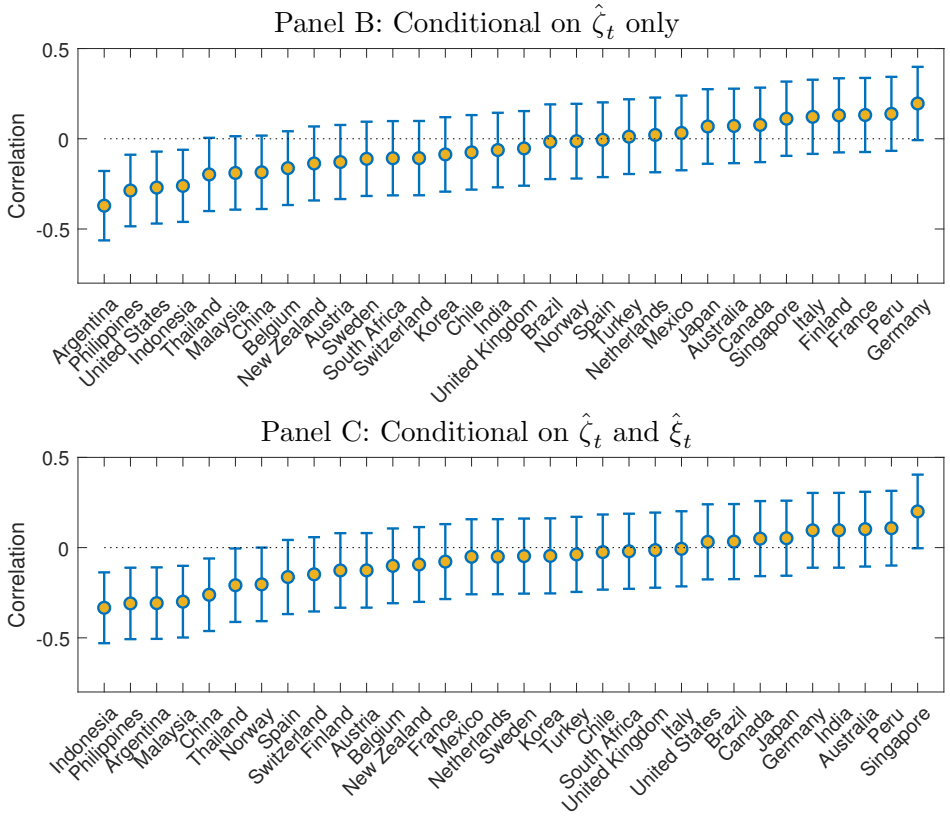
In this subsection we assess the role of the weights used in our analysis. First note that, asymptotically, the weights do not matter (Pesaran, 2006), as long as there is no dominant unit in the cross section (on the absence of dominant units in our sample, see the evidence provided in Section S.2.1). Consistently with that, we show here that our results are robust to an alternative weighting scheme.

Recall here that in our baseline analysis we used equal weights to estimate the factors, that is we assumed w_i and $\mathring{w}_i$ in Equation 23 to be $1/N$.[S6] Alternatively, one could have used PPP-GDP weights to construct the global variables and estimate the factors. Also, while in principle time-varying weights could be used, we focus here on simple weights based on the average PPP-GDP weight over the full sample period. The average PPP-GDP weights are reported in Table S.7. Clearly the US and China stand out as the largest economies in our sample.

As in the main text, Figures S.8, S.9, and S.10 report the contemporaneous correlations between the country-specific volatility and growth innovations, the impulse responses, and the forecast error variance decompositions obtained when using PPP-GDP weights for the estimation of the common factors. A comparison with the same figures in the main text shows that our results are virtually unchanged when using this alternative weighting scheme.

[S6] Remember these are the weights used to construct the global variables (as shown in Equation 23), not the weights used to aggregate results in a single average estimate, as reported in the impulse response and forecast error variance decomposition analysis (as shown in Section S.5.3).

Figure S.8 Country-specific Correlations Between Volatility and Growth Innovations (PPP-GDP Weights)



Note. Panel A displays the unconditional correlations between (log) realized stock market volatility and real GDP growth. Panel B plots the correlation between volatility and growth innovations when we condition only on $\hat{\zeta}_t$ in model (43)-(44). Panel C reports the same correlation when we condition on both $\hat{\zeta}_t$ and $\hat{\xi}_t$. The dots represent the contemporaneous correlations. The lines represent 95-percent confidence intervals. Sample period: 1993:Q1-2016:Q4.

Table S.6 Non-zero Elements of the Regularized Estimator of the Covariance Matrix

All Significant			Between-county correlations		
Country - Variable Pairs		Corr	$\hat{\varepsilon}_{it}, \hat{\varepsilon}_{jt}$	$\hat{\eta}_{it}, \hat{\eta}_{jt}$	$\hat{\varepsilon}_{it}, \hat{\eta}_{jt}$
ARG VOL	NLD VOL	-0.34		ARG,NLD	
AUS VOL	NZL VOL	0.37		AUS,NZL	
AUT GDP	PHL GDP	-0.36	AUT,PHL		
BEL VOL	ITA VOL	0.48		BEL,ITA	
BEL VOL	NLD VOL	0.56		BEL,NLD	
BEL VOL	CHE VOL	0.38		BEL,CHE	
BEL VOL	GBR VOL	0.50		BEL,GBR	
BRA VOL	MEX VOL	0.45		BRA,MEX	
CAN VOL	NOR VOL	0.44		CAN,NOR	
CHL VOL	FRA VOL	-0.35		CHL,FRA	
CHL VOL	NLD VOL	-0.35		CHL,NLD	
CHL VOL	ESP VOL	-0.36		CHL,ESP	
FIN VOL	SWE VOL	0.51		FIN,SWE	
FIN GDP	ITA GDP	0.37	FIN,ITA		
FRA VOL	DEU VOL	0.62		FRA,DEU	
FRA VOL	IND VOL	-0.38		FRA,IND	
FRA VOL	IDN VOL	-0.43		FRA,IDN	
FRA VOL	ITA VOL	0.44		FRA,ITA	
FRA VOL	MEX VOL	-0.43		FRA,MEX	
FRA VOL	NLD VOL	0.67		FRA,NLD	
FRA VOL	PHL VOL	-0.35		FRA,PHL	
FRA VOL	SGP VOL	-0.44		FRA,SGP	
FRA VOL	ESP VOL	0.58		FRA,ESP	
FRA VOL	SWE VOL	0.48		FRA,SWE	
FRA VOL	THA VOL	-0.37		FRA,THA	
FRA VOL	GBR VOL	0.60		FRA,GBR	
DEU VOL	ITA VOL	0.37		DEU,ITA	
DEU VOL	MEX VOL	-0.38		DEU,MEX	
DEU VOL	NLD VOL	0.62		DEU,NLD	
DEU VOL	ESP VOL	0.48		DEU,ESP	
DEU VOL	GBR VOL	0.37		DEU,GBR	
IDN VOL	PER GDP	-0.36			IDN,PER
IDN VOL	PHL VOL	0.38		IDN,PHL	
IDN VOL	SGP VOL	0.35		IDN,SGP	
IDN VOL	THA VOL	0.39		IDN,THA	
IDN VOL	GBR VOL	-0.36		IDN,GBR	
IDN GDP	KOR GDP	0.35	IDN,KOR		
ITA VOL	MYS VOL	-0.38		ITA,MYS	
ITA VOL	NLD VOL	0.53		ITA,NLD	
ITA VOL	PHL VOL	-0.34		ITA,PHL	
ITA VOL	SGP VOL	-0.36		ITA,SGP	
ITA VOL	ESP VOL	0.56		ITA,ESP	
ITA VOL	GBR VOL	0.39		ITA,GBR	
KOR GDP	MYS GDP	0.47	KOR,MYS		
MYS VOL	SGP VOL	0.43		MYS,SGP	
MYS VOL	SWE VOL	-0.48		MYS,SWE	
MEX VOL	NLD VOL	-0.36		MEX,NLD	
NLD VOL	PER VOL	-0.35		NLD,PER	
NLD VOL	ESP VOL	0.46		NLD,ESP	
NLD VOL	CHE VOL	0.50		NLD,CHE	
NLD VOL	GBR VOL	0.70		NLD,GBR	
NOR VOL	PER GDP	0.36			NOR,PER
PHL VOL	SGP VOL	0.48		PHL,SGP	
SGP VOL	ESP VOL	-0.39		SGP,ESP	
SWE VOL	GBR VOL	0.37		SWE,GBR	
CHE VOL	GBR VOL	0.46		CHE,GBR	
THA VOL	GBR VOL	-0.38		THA,GBR	

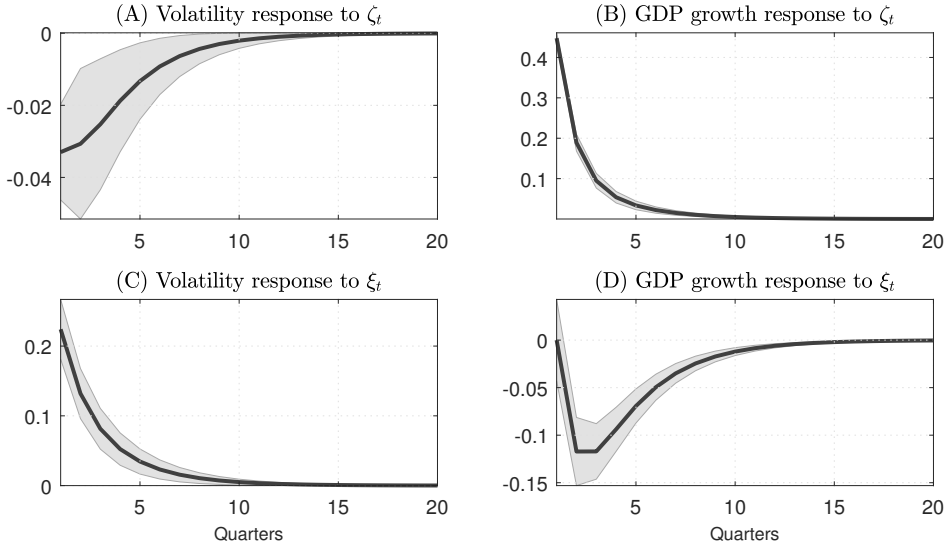
Note. Non-zero elements of the regularized error covariance matrix estimate proposed by Bailey, Pesaran, and Smith (2019). Sample period: 1993:Q1-2016:Q4.

Table S.7 PPP-GDP WEIGHTS

Argentina	1.1%	Malaysia	0.9%
Australia	1.3%	Mexico	2.6%
Austria	0.6%	Netherlands	1.1%
Belgium	0.7%	New Zealand	0.2%
Brazil	4.1%	Norway	0.4%
Canada	2.1%	Peru	0.4%
Chile	0.4%	Philippines	0.8%
China	16.0%	Singapore	0.5%
Finland	0.3%	South Africa	0.9%
France	3.7%	Spain	2.2%
Germany	5.2%	Sweden	0.6%
India	7.2%	Switzerland	0.6%
Indonesia	2.9%	Thailand	1.3%
Italy	3.3%	Turkey	1.8%
Japan	7.4%	United Kingdom	3.6%
Korea	2.2%	United States	23.7%

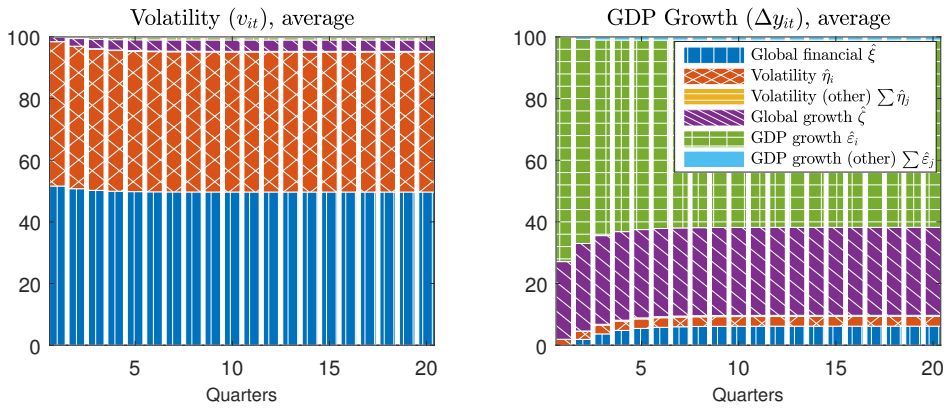
NOTE. PPP-GDP weights based on average PPP-GDP figures over the 1993:Q1-2016:Q4 period.

Figure S.9 AVERAGE COUNTRY VOLATILITY AND GROWTH RESPONSES TO REAL AND FINANCIAL FACTOR SHOCKS (PPP-GDP WEIGHTS)



NOTE. Average impulse responses to one-standard deviation real and financial shocks, $\hat{\zeta}_t$ and $\hat{\xi}_t$. The solid lines are the PPP-GDP weighted averages of the country-specific responses. The shaded areas are two standard deviations confidence intervals. The vertical axis is in percent, the horizontal axis is in quarters. Sample period: 1993:Q1-2016:Q4.

Figure S.10 Forecast Error Variance Decomposition of Country-specific Shocks (PPP-GDP Weights)

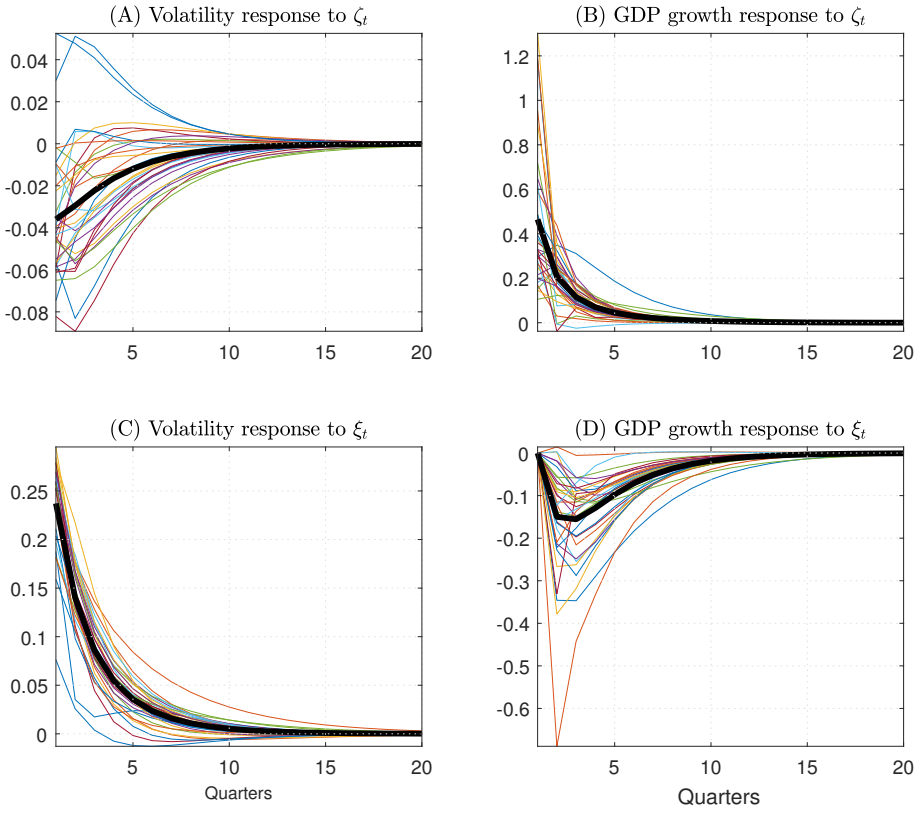


Note. Block-diagonal covariance matrix, with Cholesky decomposition of within-country covariance. Average across countries with GDP-PPP weights. $\hat{\xi}$ is common financial shock (blue area with vertical lines); $\hat{\eta}_i$ is country-specific volatility shock (red area with crosses); $\sum \hat{\eta}_j$ is the sum of the contribution of the volatility shocks in the remaining countries (yellow area with horizontal lines); $\hat{\zeta}$ is common growth shock (purple area with diagonal lines); $\hat{\varepsilon}_i$ is country-specific GDP growth shock (green areas with squares); $\sum \hat{\varepsilon}_j$ is the sum of the contributions of the GDP growth shocks in the remaining countries (light blue areas with no pattern). The vertical axis is in percent, the horizontal axis is in quarters. Sample period: 1993:Q1-2016:Q4.

S.3 Country-specific Results

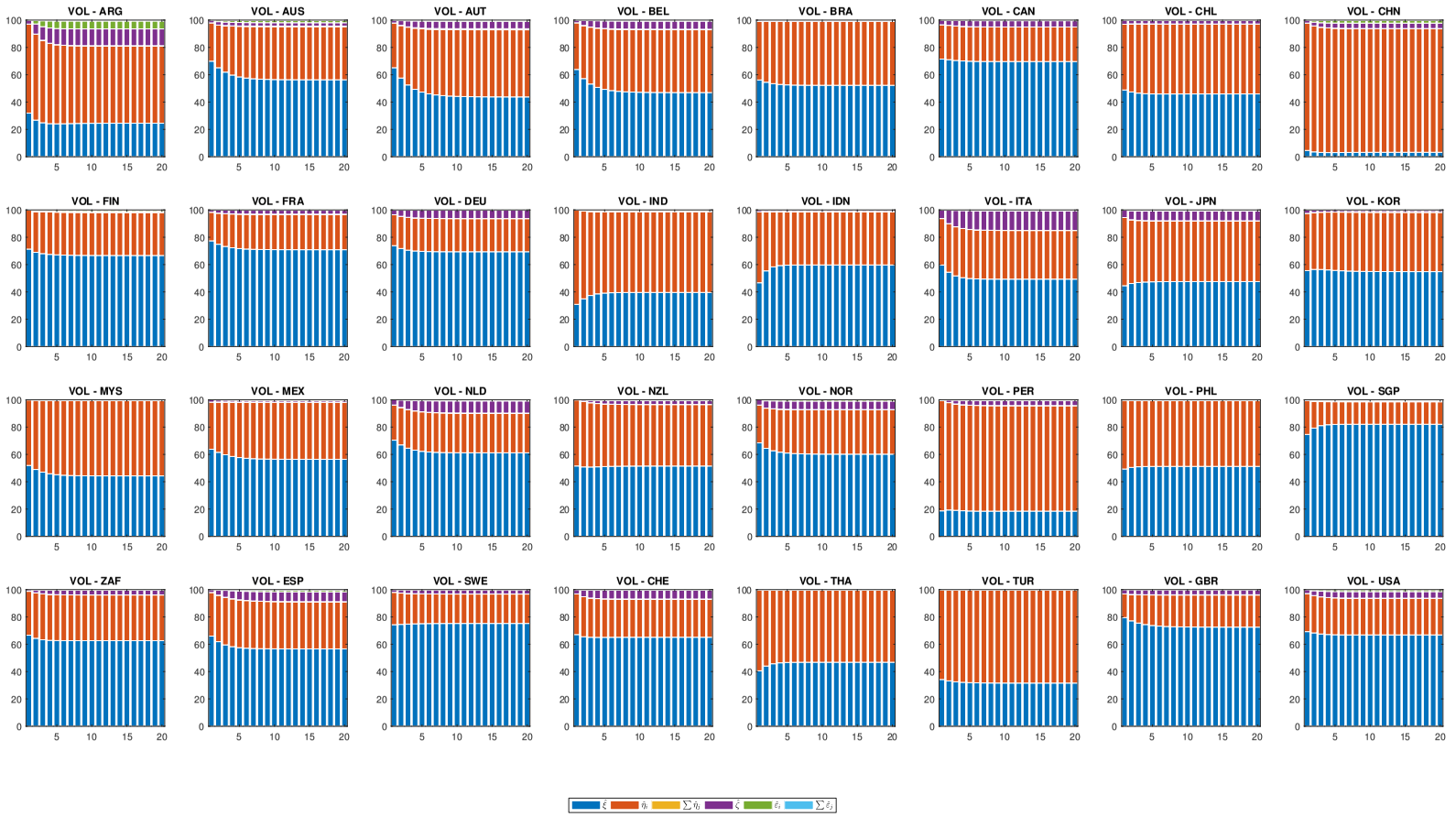
In this section we report selected country-specific results, namely the individual country impulse response functions and forecast error variance decompositions. Figure S.11 plots the country-specific impulse response of volatility and growth to a positive, one-standard-deviation shock to the global shocks, $\hat{\zeta}_t$ and $\hat{\xi}_t$. We can see from Figure S.11 that for most countries the impulse responses have a very similar profile. Figures S.12 to S.17 report forecast error variance decompositions for each country, for both volatility and growth, computed with different assumptions on the covariance matrix of the volatility and growth innovations. As can be seen the estimates are very similar across countries and for all the three schemes assumed for the error covariances.

Figure S.11 Country-specific Volatility and Growth Impulse Responses to Common Real and Financial Shocks



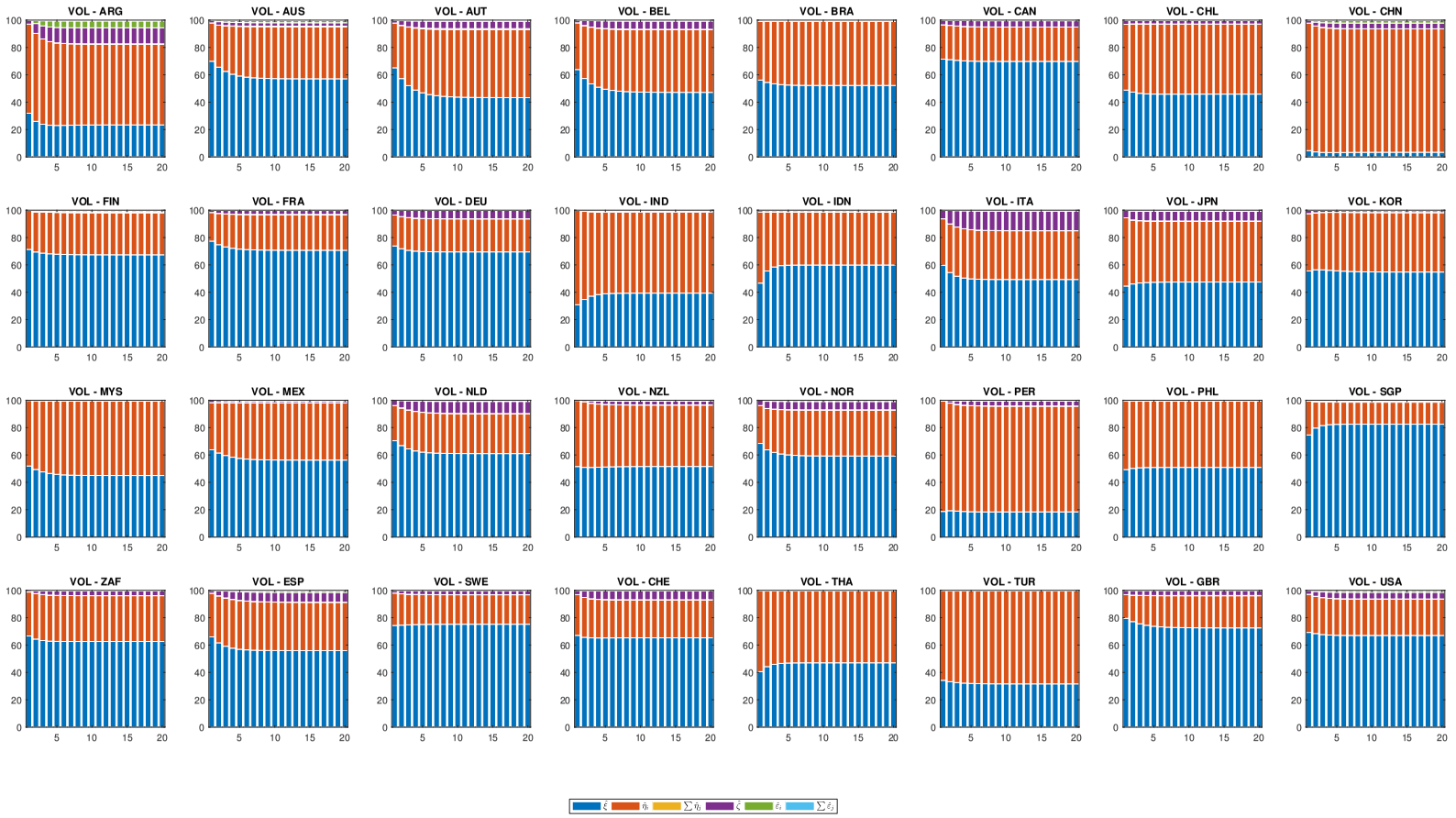
Note. One standard deviation shocks to $\hat{\zeta}_t$ and $\hat{\xi}_t$. Thin lines are individual country responses. The solid lines are the PPP-GDP weighted averages, as the ones reported in the main text. The horizontal axis is in quarters. Sample period: 1993:Q1-2016:Q4.

Figure S.12 Forecast Error Variance Decomposition of Country-Specific Volatility Shocks - Diagonal Error Covariance Matrix



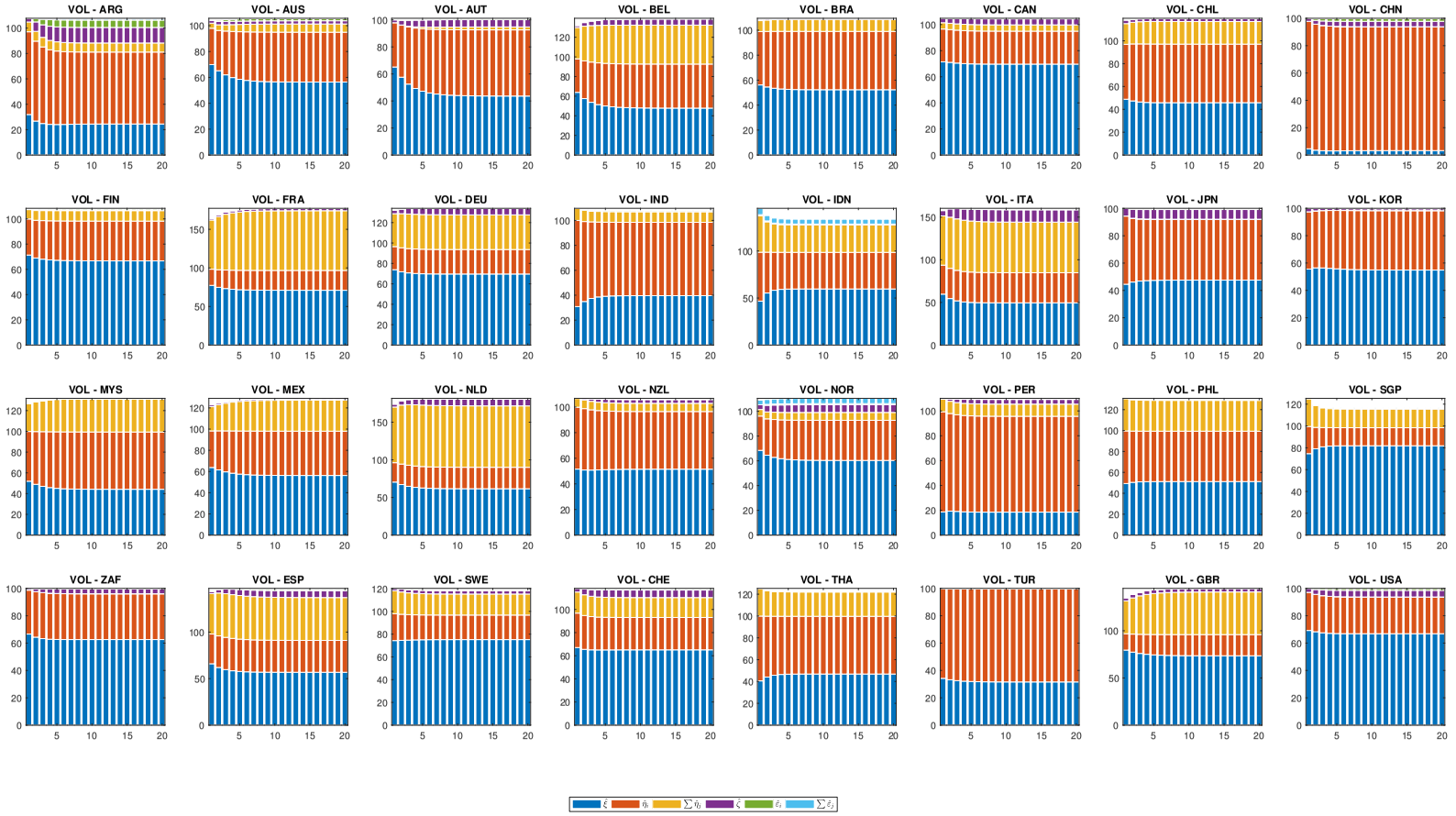
Note. $\hat{\xi}$ is common financial shock (blue area); $\hat{\eta}_i$ is country-specific volatility shock (red area); $\sum \hat{\eta}_j$ is the sum of the contribution of the volatility shocks in the remaining countries (yellow area); $\hat{\zeta}$ is common growth shock (purple area); $\hat{\varepsilon}_i$ is country-specific GDP growth shock (green areas); $\sum \hat{\varepsilon}_j$ is the sum of the contributions of the GDP growth shocks in the remaining countries (light blue areas). The vertical axis is in percent, the horizontal axis is in quarters. Sample period: 1993:Q1-2016:Q4.

Figure S.13 Forecast Error Variance Decomposition of Country-Specific Volatility Shocks - Block Diagonal Error Covariance Matrix



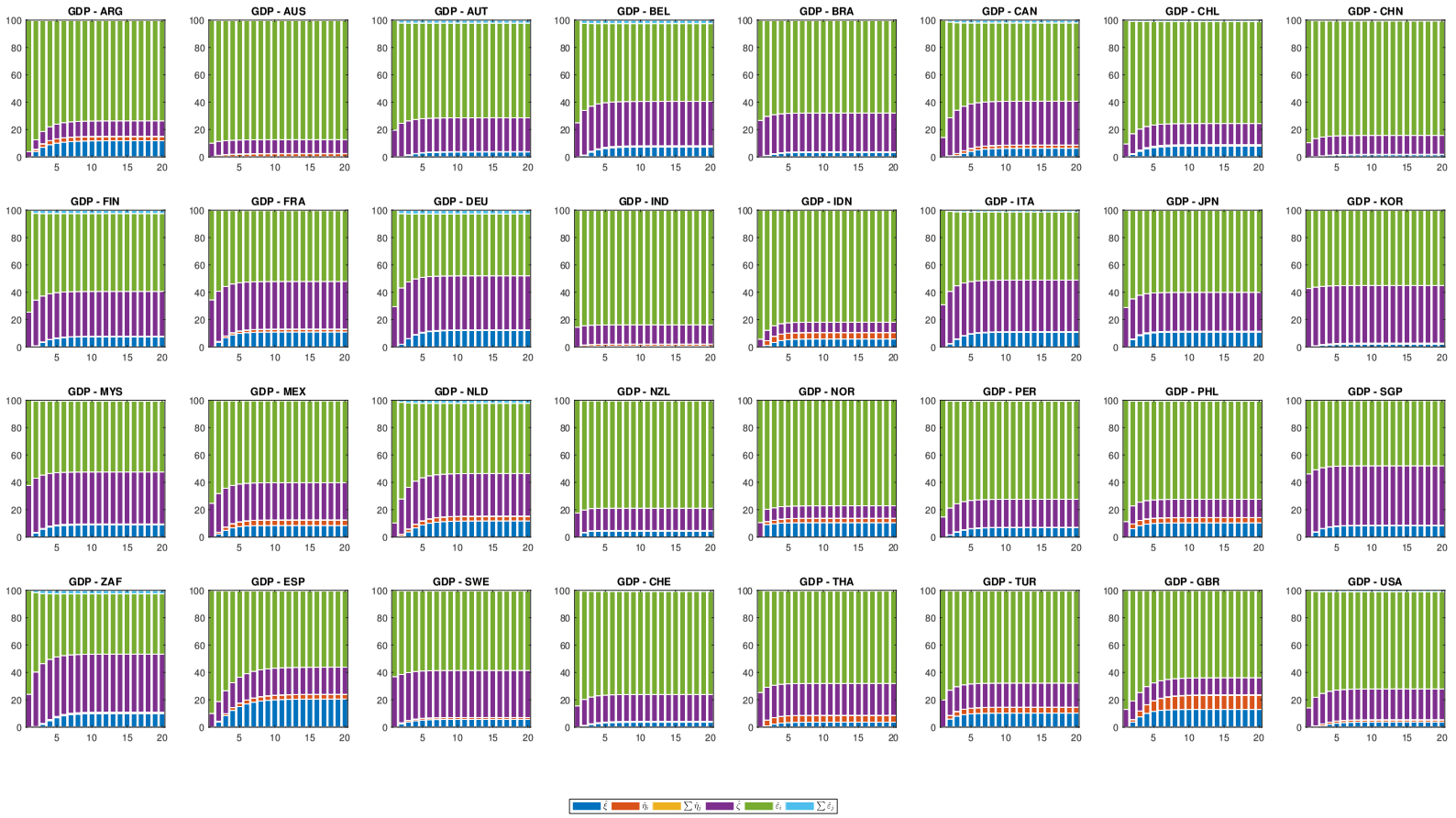
Note. $\hat{\xi}$ is common financial shock (blue area); $\hat{\eta}_i$ is country-specific volatility shock (red area); $\sum \hat{\eta}_j$ is the sum of the contribution of the volatility shocks in the remaining countries (yellow area); $\hat{\zeta}$ is common growth shock (purple area); $\hat{\varepsilon}_i$ is country-specific GDP growth shock (green areas); $\sum \hat{\varepsilon}_j$ is the sum of the contributions of the GDP growth shocks in the remaining countries (light blue areas). The vertical axis is in percent, the horizontal axis is in quarters. Sample period: 1993:Q1-2016:Q4.

Figure S.14 Generalized Forecast Error Variance Decomposition of Country-Specific Volatility Shocks - Regularized Estimation of Full Error Covariance Matrix



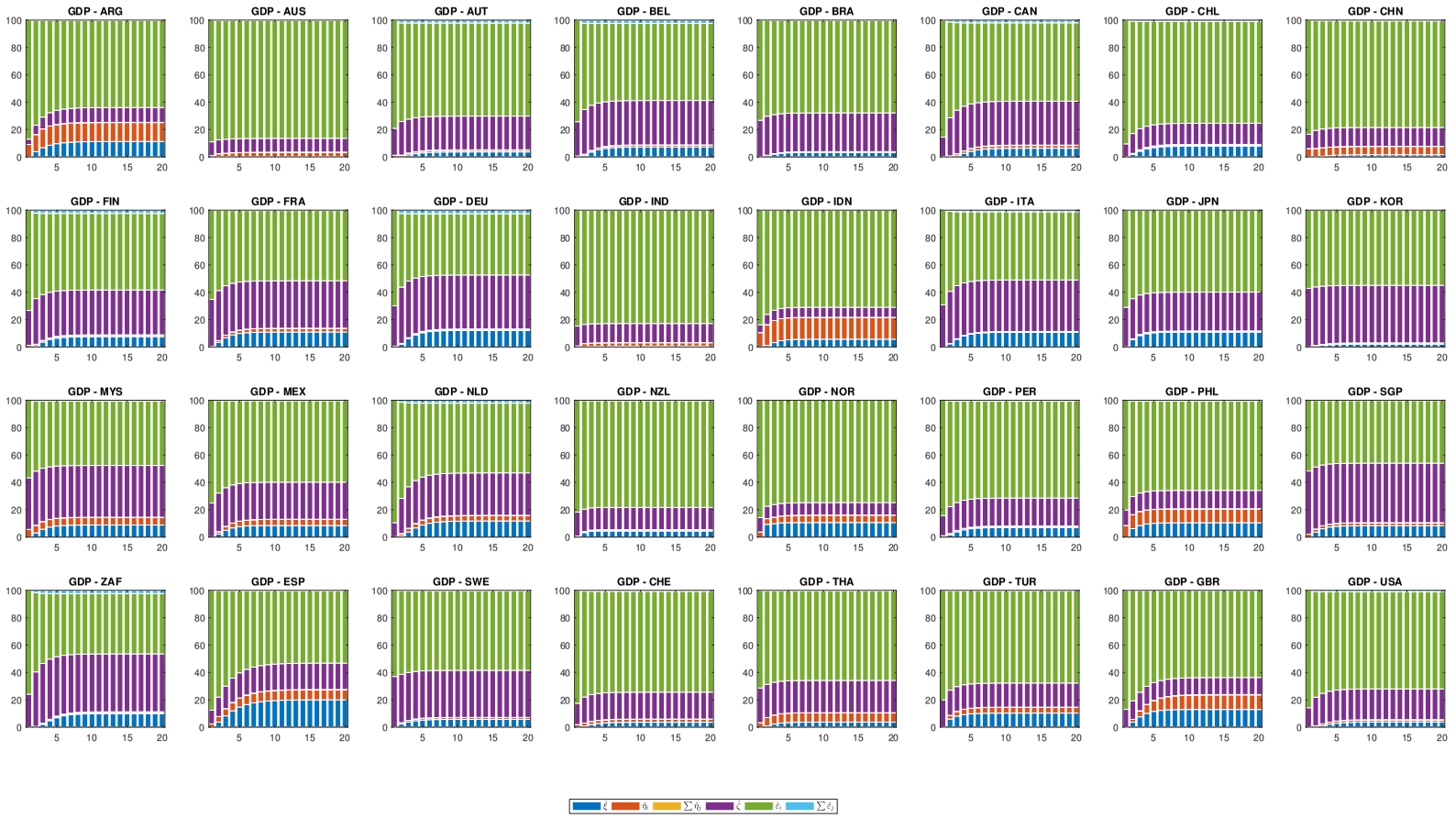
Note. $\hat{\xi}$ is common financial shock (blue area); $\hat{\eta}_i$ is country-specific volatility shock (red area); $\sum \hat{\eta}_j$ is the sum of the contribution of the volatility shocks in the remaining countries (yellow area); $\hat{\zeta}$ is common growth shock (purple area); $\hat{\varepsilon}_i$ is country-specific GDP growth shock (green areas); $\sum \hat{\varepsilon}_j$ is the sum of the contributions of the GDP growth shocks in the remaining countries (light blue areas). The vertical axis is in percent, the horizontal axis is in quarters. Sample period: 1993:Q1-2016:Q4.

Figure S.15 Forecast Error Variance Decomposition of Country-Specific Growth Shocks - Diagonal Error Covariance Matrix



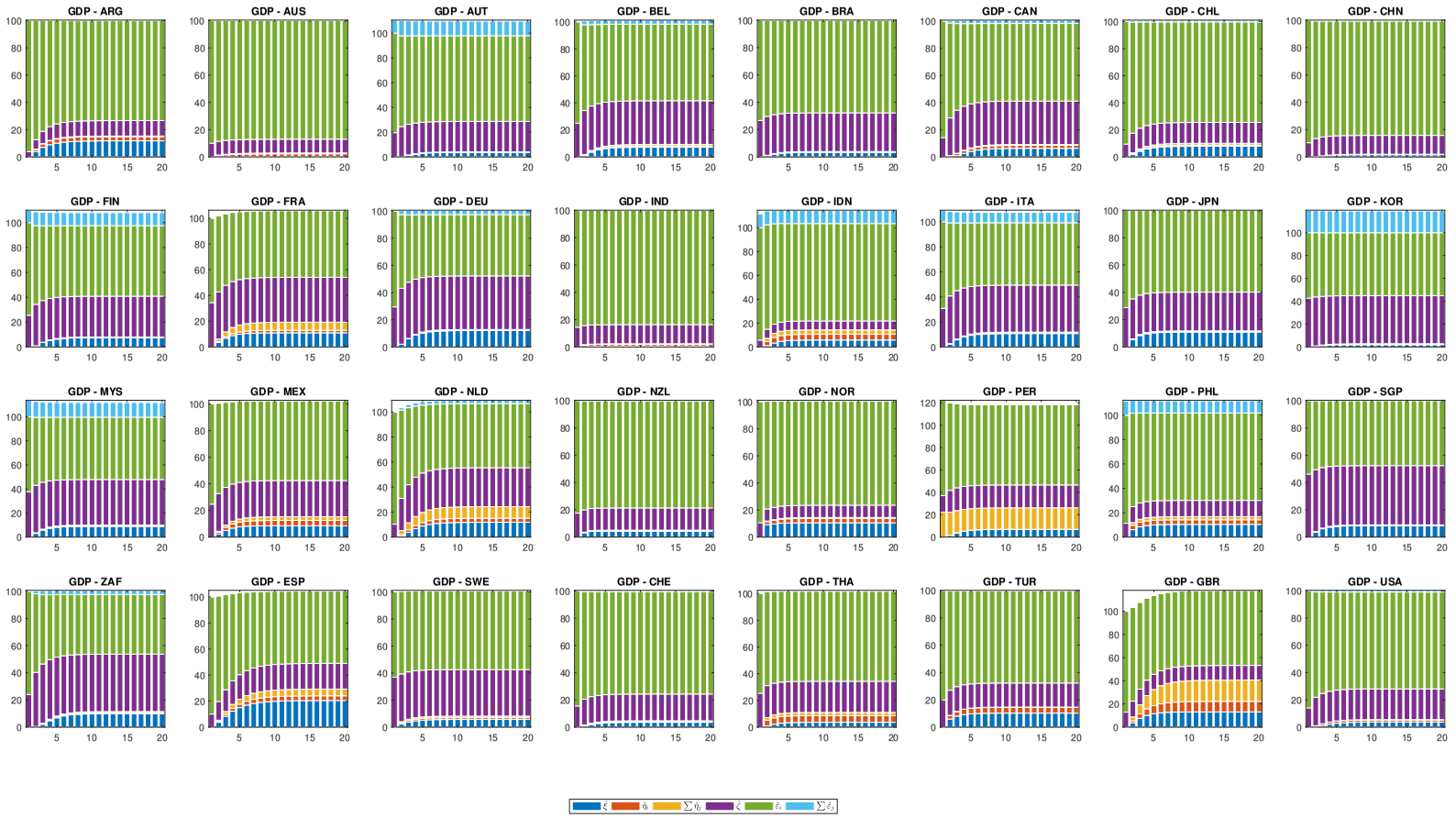
Note. $\hat{\xi}$ is common financial shock (blue area); $\hat{\eta}_i$ is country-specific volatility shock (red area); $\sum \hat{\eta}_j$ is the sum of the contribution of the volatility shocks in the remaining countries (yellow area); $\hat{\zeta}$ is common growth shock (purple area); $\hat{\varepsilon}_i$ is country-specific GDP growth shock (green areas); $\sum \hat{\varepsilon}_j$ is the sum of the contributions of the GDP growth shocks in the remaining countries (light blue areas). The vertical axis is in percent, the horizontal axis is in quarters. Sample period: 1993:Q1-2016:Q4.

Figure S.16 Forecast Error Variance Decomposition of Country-Specific Growth Shocks - Block Diagonal Error Covariance Matrix



Note. $\hat{\xi}$ is common financial shock (blue area); $\hat{\eta}_i$ is country-specific volatility shock (red area); $\sum \hat{\eta}_j$ is the sum of the contribution of the volatility shocks in the remaining countries (yellow area); $\hat{\zeta}$ is common growth shock (purple area); $\hat{\varepsilon}_i$ is country-specific GDP growth shock (green areas); $\sum \hat{\varepsilon}_j$ is the sum of the contributions of the GDP growth shocks in the remaining countries (light blue areas). The vertical axis is in percent, the horizontal axis is in quarters. Sample period: 1993:Q1-2016:Q4.

Figure S.17 Generalized Forecast Error Variance Decomposition of Country-Specific Growth Shocks - Regularized Estimation of Full Error Covariance Matrix



Note. $\hat{\xi}$ is common financial shock (blue area); $\hat{\eta}_i$ is country-specific volatility shock (red area); $\sum \hat{\eta}_j$ is the sum of the contribution of the volatility shocks in the remaining countries (yellow area); $\hat{\zeta}$ is common growth shock (purple area); $\hat{\varepsilon}_i$ is country-specific GDP growth shock (green areas); $\sum \hat{\varepsilon}_j$ is the sum of the contributions of the GDP growth shocks in the remaining countries (light blue areas). The vertical axis is in percent, the horizontal axis is in quarters. Sample period: 1993:Q1-2016:Q4.

S.4 Realized Volatility versus Cross-sectional Dispersion

As noted in the paper, if we consider a panel of country-specific equities (e.g. of firms or sectors within a country), a different measure of uncertainty can be computed as the cross-sectional dispersion of equity prices. In this section we show that this concept is closely related to the realized volatility measure we consider. To illustrate the point with the data that we use in our application, we derive results at the 'country-specific versus world level' rather than 'firm-specific versus country level'.[S7] Specifically, we compare the cross-sectional dispersion of equity returns across countries with the realized volatility of 'world' equity returns.

Define the *daily* cross-country dispersion of equity returns as:

$$\sigma_{cdt} = \sqrt{D_t^{-1} \sum_{\tau=1}^{D_t} \sum_{i=1}^{N} w_i \left[r_{it}(\tau) - \bar{r}_t(\tau) \right]^2}, \tag{S1}$$

and the *daily* realized volatility of world equity returns as:

$$\sigma_{rvt} = \sqrt{D_t^{-1} \sum_{i=1}^{N} \sum_{\tau=1}^{D_t} w_i \left[r_{it}(\tau) - \bar{r}_{it} \right]^2}, \tag{S2}$$

where $r_{it}(\tau) = \Delta \ln P_{it}(\tau)$, $P_{it}(\tau)$ is the price of equity at close of day τ of quarter t in country i, $\bar{r}_t(\tau) = \sum_{i=1}^{N} w_i r_{it}(\tau)$ is the weighted cross section average of price changes during day τ in quarter t, and $\bar{r}_{it} = D_t^{-1} \sum_{\tau=1}^{D_t} r_{it}(\tau)$ is the average daily rate of price change of country i^{th} equity return over the quarter t, and D_t is the number of trading days in quarter t; and w_i is the weight attached to country i. To establish the relation between these two measures it is easier to work with their squares:

$$\sigma_{rvt}^2 = D_t^{-1} \sum_{i=1}^{N} \sum_{\tau=1}^{D_t} w_i \left[r_{it}(\tau) - \bar{r}_{it} \right]^2, \quad \sigma_{cdt}^2 = D_t^{-1} \sum_{\tau=1}^{D_t} \sum_{i=1}^{N} w_i \left[r_{it}(\tau) - \bar{r}_t(\tau) \right]^2.$$

Note also that

$$\sigma_{rvt}^2 = D_t^{-1} \sum_{i=1}^{N} \sum_{\tau=1}^{D_t} w_i r_{it}^2(\tau) - \sum_{i=1}^{N} w_i \bar{r}_{it}^2,$$

and

$$\sigma_{cdt}^2 = D_t^{-1} \sum_{\tau=1}^{D_t} \sum_{i=1}^{N} w_i r_{it}^2(\tau) - \sum_{i=1}^{N} w_i \left(D_t^{-1} \sum_{\tau=1}^{D_t} \bar{r}_t^2(\tau) \right).$$

Hence, since $\sum_{i=1}^{N} w_i = 1$, it follows that

$$\sigma_{cdt}^2 - \sigma_{rvt}^2 = \sum_{i=1}^{N} w_i \bar{r}_{it}^2 - D_t^{-1} \sum_{\tau=1}^{D_t} \bar{r}_t^2(\tau).$$

Suppose now that daily returns have the following single-factor structure:[S8]

$$r_{it}(\tau) = \beta_i f_t(\tau) + \varepsilon_{it}(\tau),$$

[S7] Our analysis holds at the firm-specific versus country level as well.

[S8] The analysis readily extends to more general multiple factor settings.

where the factor is strong in the sense that (Bailey et al. (2016))

$$\lim_{N\to\infty}\sum_{i=1}^{N} w_i\beta_i = \bar{\beta} \neq 0, \text{ and } \lim_{N\to\infty}\sum_{i=1}^{N} w_i\beta_i^2 = \sigma_\beta^2 + \bar{\beta}^2 > 0.$$

The idiosyncratic components, $\varepsilon_{it}(\tau)$, are assumed to be independently distributed from $\beta_i f_t(\tau)$, cross-sectionally weakly correlated, and serially uncorrelated with zero means and finite variances. Also let:

$$\lim_{D_t\to\infty} D_t^{-1}\sum_{\tau=1}^{D_t} f_t^2(\tau) = h_{f_t}^2.$$

We now note that

$$\begin{aligned}\sum_{i=1}^{N} w_i\bar{r}_{it}^2 &= \left(\sum_{i=1}^{N} w_i\beta_i^2\right)\bar{f}_t^2 + \left(\sum_{i=1}^{N} w_i\bar{\varepsilon}_{it}^2\right) + 2\left(\sum_{i=1}^{N} w_i\beta_i\bar{\varepsilon}_{it}\right)\bar{f}_t \\ &= \left(\sigma_\beta^2 + \bar{\beta}^2\right)\bar{f}_t^2 + O_p\left(D_t^{-1/2}\right) + O_p\left(N^{-1/2}\right),\end{aligned}$$

where $\bar{f}_t = D_t^{-1}\sum_{\tau=1}^{D_t} f_t(\tau)$, and $\bar{\varepsilon}_{it} = D_t^{-1}\sum_{\tau=1}^{D_t}\varepsilon_{it}(\tau)$. Also

$$\begin{aligned}D_t^{-1}\sum_{\tau=1}^{D_t}\bar{r}_t^2(\tau) &= D_t^{-1}\sum_{\tau=1}^{D_t}\left[\bar{\beta}f_t(\tau) + \bar{\varepsilon}_t(\tau)\right]^2 \\ &= \bar{\beta}^2\left[D_t^{-1}\sum_{\tau=1}^{D_t} f_t^2(\tau)\right] + D_t^{-1}\sum_{\tau=1}^{D_t}\bar{\varepsilon}_t^2(\tau) + 2D_t^{-1}\sum_{\tau=1}^{D_t}\bar{\beta}\bar{\varepsilon}_t(\tau)f_t(\tau) \\ &= \bar{\beta}^2 h_{f_t}^2 + O_p\left(N^{-1/2}\right) + O_p\left(D_t^{-1/2}\right).\end{aligned}$$

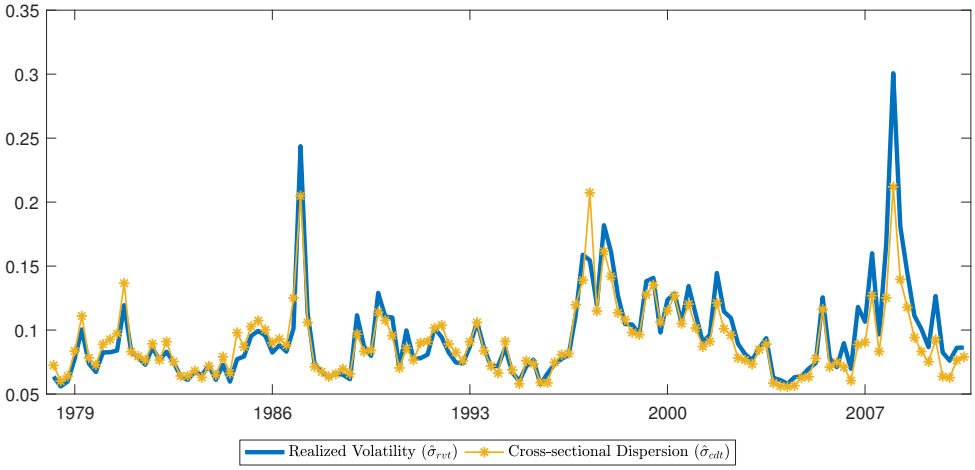
Hence

$$\begin{aligned}\sigma_{cdt}^2 - \sigma_{rvt}^2 &= \left(\sigma_\beta^2 + \bar{\beta}^2\right)\bar{f}_t^2 - \bar{\beta}^2 h_{f_t}^2 + O_p\left(N^{-1/2}\right) + O_p\left(D_t^{-1/2}\right) \\ &= \sigma_\beta^2\bar{f}_t^2 - \bar{\beta}^2\sigma_{f_t}^2 + O_p\left(N^{-1/2}\right) + O_p\left(D_t^{-1/2}\right).\end{aligned}$$

where $\sigma_{f_t}^2 = \left(h_{f_t}^2 - \bar{f}_t^2\right) \geq 0$, is the variance of the common factor. This expression shows that, under fairly general assumptions (and for N and D_t sufficiently large) we would expect the cross-sectional dispersion measure to be closely related to asset-specific measures of realized volatility when the factor loadings, β_i, are not too dispersed across countries. The results also show that the relative magnitudes of the cross section dispersion and realized volatility depends on the relative values of $\sigma_\beta^2\bar{f}_t^2$ and $\bar{\beta}^2\sigma_{f_t}^2$.

Figure S.18 compares world realized volatility (σ_{rvt}, light thick line) and cross-sectional dispersion (σ_{cdt}, dark thin line), computed as in equations (S2) and (S1), respectively, with equal weights. Their sample correlation over the 1979:Q1 to 2016:Q4 period is 0.92. Figure S.18 suggests that the two measures are very closely related, which is in line with the evidence provided by Bloom et al. (2012).

Figure S.18 Realized Volatility and Cross-sectional Dispersion



Note. World realized volatility of equity returns ($\hat{\sigma}_{rvt}$) and cross-sectional dispersion of equity returns across countries ($\hat{\sigma}_{cdt}$), computed as in equations (S2) and (S1), respectively. Both measures are expressed at quarterly rates and computed over the 1979:Q2-2016:Q4 period.

S.5 Computing Impulse Responses and Error Variance Decompositions

Consider the factor-augmented country-specific VAR models augmented with lagged cross section averages, $\bar{\mathbf{z}}_{\omega,t-\ell}$, for $\ell = 1, 2, ..., p$ as in equations (43)-(44) in the main text:

$$\mathbf{z}_{it} = \mathbf{\Phi}_i \mathbf{z}_{i,t-1} + \sum_{\ell=1}^{p} \mathbf{D}_{i\ell} \bar{\mathbf{z}}_{\omega,t-\ell} + \boldsymbol{\beta}_i \boldsymbol{\delta}_t + \boldsymbol{\vartheta}_{it}, \text{ for } i = 1, 2, ..., N, \tag{S1}$$

where:

$$\mathbf{D}_{i\ell} = \begin{pmatrix} d_{1v,i\ell} & d_{2v,i\ell} \\ d_{1\Delta y,i\ell} & d_{2\Delta y,i\ell} \end{pmatrix}, \boldsymbol{\beta}_i = \begin{pmatrix} \beta_{i,11} & \beta_{i,12} \\ \beta_{i,21} & 0 \end{pmatrix}, \boldsymbol{\delta}_t = \begin{pmatrix} \zeta_t \\ \xi_t \end{pmatrix}, \boldsymbol{\vartheta}_{it} = \begin{pmatrix} \varepsilon_{it} \\ \eta_{it} \end{pmatrix}.$$

Intercepts are omitted to simplify the exposition. Note also that $\bar{\mathbf{z}}_{\omega,t} = \sum_{i=1}^{N} w_i \Delta \mathbf{z}_{it} = \mathbf{W}\mathbf{z}_t$, where $\mathbf{z}_t = (\mathbf{z}'_{1t}, \mathbf{z}'_{2t}, ..., \mathbf{z}'_{Nt})'$, and $\mathbf{W}$ is a $2 \times 2N$ matrix of weights. Stacking the VARs in (S1) over i we obtain:

$$\mathbf{z}_t = \mathbf{\Phi}\mathbf{z}_{t-1} + \sum_{\ell=1}^{p} \mathbf{D}_\ell \mathbf{W}\mathbf{z}_{t-\ell} + \boldsymbol{\beta}\boldsymbol{\delta}_t + \boldsymbol{\vartheta}_t, \tag{S2}$$

where $\boldsymbol{\vartheta}_t = (\boldsymbol{\vartheta}'_{1t}, \boldsymbol{\vartheta}'_{2t}, ..., \boldsymbol{\vartheta}'_{Nt})'$ and:

$$\mathbf{\Phi} = \begin{pmatrix} \mathbf{\Phi}_1 & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{\Phi}_2 & \cdots & \mathbf{0} \\ \vdots & \vdots & \cdots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{\Phi}_N \end{pmatrix}, \quad \mathbf{D}_\ell = \begin{pmatrix} \mathbf{D}_{1,\ell} \\ \mathbf{D}_{2,\ell} \\ \vdots \\ \mathbf{D}_{N,\ell} \end{pmatrix}, \quad \boldsymbol{\beta} = \begin{pmatrix} \boldsymbol{\beta}_1 \\ \boldsymbol{\beta}_2 \\ \vdots \\ \boldsymbol{\beta}_N \end{pmatrix}.$$

The high-dimensional VAR in (S2) can now be written as a standard FAVAR(p) model in $2N$ variables:

$$\mathbf{z}_t = (\mathbf{\Phi} + \mathbf{D}_1\mathbf{W})\,\mathbf{z}_{t-1} + \sum_{\ell=2}^{p} \mathbf{D}_\ell \mathbf{W}\mathbf{z}_{t-\ell} + \boldsymbol{\beta}\boldsymbol{\delta}_t + \boldsymbol{\vartheta}_t. \tag{S3}$$

For example, when $p = 1$ we have the FAVAR(1):

$$\mathbf{z}_t = (\mathbf{I}_{2N} - \mathbf{\Psi}_1\mathbf{L})^{-1}(\boldsymbol{\beta}\boldsymbol{\delta}_t + \boldsymbol{\vartheta}_t),$$

where $\mathbf{\Psi}_1 = \mathbf{\Phi} + \mathbf{D}_1\mathbf{W}$ and

$$\mathbf{z}_t = (\mathbf{I}_{2N} - \mathbf{\Psi}_1\mathbf{L})^{-1}\boldsymbol{\beta}\boldsymbol{\delta}_t + (\mathbf{I} - \mathbf{\Psi}_1\mathbf{L})^{-1}\boldsymbol{\vartheta}_t.$$

Note that by construction $\boldsymbol{\delta}_t$ and $\boldsymbol{\vartheta}_t$ are orthogonal, and for sufficiently large p, they are serially uncorrelated. Hence, the impulse response of shocks to elements of $\boldsymbol{\delta}_t$ and $\boldsymbol{\vartheta}_t$ can be computed using the following moving average representation:

$$\mathbf{z}_t = \sum_{n=0}^{\infty} \mathbf{A}_n \boldsymbol{\delta}_{t-n} + \sum_{n=0}^{\infty} \mathbf{C}_n \boldsymbol{\vartheta}_{t-n}, \tag{S4}$$

where $\mathbf{A}_n = \mathbf{\Psi}_1^n\boldsymbol{\beta}$, and $\mathbf{C}_n = \mathbf{\Psi}_1^n$, for $n = 0, 1, 2, ...$

S.5.1 Responses to Common and Country-specific Shocks

Let $\mathfrak{e}_i$ be a selection vector such that $\mathfrak{e}_i'\mathbf{z}_t$ picks the i^{th} element of $\mathbf{z}_t$. Also let $\mathbf{s}_f = (1, 0)'$ and $\mathbf{s}_g = (0, 1)'$, the vectors that select ζ_t and ξ_t from $\boldsymbol{\delta}_t$, namely:

$$\mathbf{s}_f'\boldsymbol{\delta}_t \equiv \zeta_t, \quad \mathbf{s}_g'\boldsymbol{\delta}_t \equiv \xi_t. \tag{S5}$$

Recall now that ζ_t and ξ_t have zero means, unit variances and are orthogonal to each other. Then the impulse responses to a positive unit shock to ζ_t or ξ_t are given by:

$$IR_{i,\zeta,n} = \mathfrak{e}_i'\mathbf{A}_n\mathbf{s}_f \quad \text{and} \quad IR_{i,\xi,n} = \mathfrak{e}_i'\mathbf{A}_n\mathbf{s}_g \quad \text{for } n = 0, 1, 2, ..., \tag{S6}$$

where $\mathbf{A}_n$ is given by the moving average representation, (S4)

To derive impulse response functions for country-specific shocks, namely the individual elements of $\boldsymbol{\vartheta}_t$, we need to make assumptions about the correlation between volatility and growth innovations within each country and across countries. Since the elements of $\boldsymbol{\vartheta}_t$ are weakly correlated across countries, they have some, but limited correlations across countries (see Figure 8). We also documented that, conditional on the common shocks ζ_t and ξ_t, the country-specific correlation of volatility and growth innovations are statistically insignificant for all except for four countries.

As a first order approximation, therefore, we will assume that the covariance matrix of $\boldsymbol{\vartheta}_t$ in (S3) is diagonal. Under this assumption, the impulse response function of a positive, unit shock to the j^{th} element of $\boldsymbol{\vartheta}_t$ on the the i^{th} element of $\mathbf{z}_t$ is given by:

$$IR_{i,\vartheta_j,n} = \sqrt{\hat{\omega}_{jj}}\,\mathfrak{e}_i'\mathbf{C}_n\mathfrak{e}_j\ , \tag{S7}$$

where $\mathbf{C}_n$ is given by the moving average representation, (S4), $\hat{\omega}_{jj}$ is the (estimate) of the variance of the j^{th} country-specific shock and $\mathfrak{e}_j$ is a selection vector such that $\mathfrak{e}_j'\mathbf{z}_t$ picks the j^{th} element of $\mathbf{z}_t$.

The above impulse responses can be compared to the generalized impulse responses of Pesaran

and Shin (1998). The latter are given by:

$$GIR_{i,\vartheta_j,n} = \frac{\mathfrak{e}_i' \mathbf{C}_n \hat{\Omega} \mathfrak{e}_j}{\sqrt{\hat{\omega}_{jj}}}, \tag{S8}$$

where $\hat{\mathbf{\Omega}} = (\hat{\omega}_{ij})$ is the estimate of the covariance of $\boldsymbol{\vartheta}_t$. The generalized impulse responses allow for non-zero correlations across the idiosyncratic errors. The two sets of impulse responses coincide if the covariance matrix of $\boldsymbol{\vartheta}_t$ is diagonal.

S.5.2 Forecast Error Variance Decompositions

Traditionally, the forecast error variance decomposition of a VAR model is performed on a set of orthogonalized shocks, whereby the contribution of the j^{th} orthogonalized innovation to the mean square error of the n-step ahead forecast of the model is calculated. In our empirical application this is not the case as —even if the country-specific volatility and growth innovations η_{it} and ε_{it} are weakly correlated across countries— some pairs of innovations can still display some non-zero correlation. An alternative approach is to compute Generalized Forecast Error Variance Decompositions (GVD) of Pesaran and Shin (1998). The Generalized Forecast Error Variance Decompositions consider the proportion of the variance of the n-step forecast errors of the endogenous variables that is explained by conditioning on the non-orthogonalized shocks, while explicitly allowing for the contemporaneous correlations between these shocks and the shocks to the other equations in the system.

Let $GVD_{i,\zeta,n}$ and $GVD_{i,\xi,n}$ be the share of the n-step ahead forecast error variance of the i^{th} variable in $\mathbf{z}_t$ that is accounted for by ζ_t and ξ_t, respectively, and $GVD_{i,j}$ the variance share of a generic country-specific shock, then:

$$GVD_{i,\zeta,n} = \frac{\sum_{\ell=0}^{n} \left(\mathfrak{e}_i' \mathbf{A}_\ell \mathbf{s}_f\right)^2}{\sum_{\ell=0}^{n} \mathfrak{e}_i' \mathbf{A}_\ell \mathbf{A}_\ell' \mathfrak{e}_i + \sum_{\ell=0}^{n} \mathfrak{e}_i' \mathbf{C}_\ell \hat{\mathbf{\Omega}} \mathbf{C}_\ell' \mathfrak{e}_i}, \quad n = 1, 2, ..., H, \tag{S9}$$

$$GVD_{i,\xi,n} = \frac{\sum_{\ell=0}^{n} \left(\mathfrak{e}_i' \mathbf{A}_\ell \mathbf{s}_g\right)^2}{\sum_{\ell=0}^{n} \mathfrak{e}_i' \mathbf{A}_\ell \mathbf{A}_\ell' \mathfrak{e}_i + \sum_{\ell=0}^{n} \mathfrak{e}_i' \mathbf{C}_\ell \hat{\mathbf{\Omega}} \mathbf{C}_\ell' \mathfrak{e}_i}, \quad n = 1, 2, ..., H, \tag{S10}$$

$$GVD_{i,j,n} = \frac{\hat{\omega}_{jj}^{-1} \sum_{\ell=0}^{n} \left(\mathfrak{e}_i' \mathbf{C}_\ell \hat{\mathbf{\Omega}} \mathfrak{e}_j\right)^2}{\sum_{\ell=0}^{n} \mathfrak{e}_i' \mathbf{A}_\ell \mathbf{A}_\ell' \mathfrak{e}_i + \sum_{\ell=0}^{n} \mathfrak{e}_i' \mathbf{C}_\ell \hat{\mathbf{\Omega}} \mathbf{C}_\ell' \mathfrak{e}_i}, \quad j = 1, 2, ..., 2N, \quad n = 1, 2, ..., H; \tag{S11}$$

Note that the different assumptions we make on the covariance matrix of all country-specific shocks, $\hat{\mathbf{\Omega}}$, have implications for the error variance decompositions. Specifically, when we assume that (i) $\hat{\mathbf{\Omega}}$ is diagonal or (ii) $\hat{\mathbf{\Omega}}$ is block-diagonal with Cholesky-orthogonalized blocks, the relative importance of shocks to country-specific volatility and growth for all countries (η_{it} and ε_{it}, for $j = 1, 2, ..., 2N$) and shocks to the two common factors ζ_t and ξ_t, is easily characterized as $VD_{i,\zeta,n} + VD_{i,\xi,n} + \sum_{j=1}^{2N} VD_{i,j,n} = 1$. That is the GVD formula coincides with the standard VD formula. In contrast, when we consider an unrestricted covariance matrix $\hat{\mathbf{\Omega}}$, the sum of the variance shares does not necessarily add up to 1.

S.5.3 Average Impulse Responses and Forecast Error Variance Decompositions

As a summary measure of the effects of shocks to the common factors we report the following average measures. Denote the impulse response (or forecast error variance decomposition) of a particular shock on the j^{th} variable in country i at horizon n by $X_{i,j,n}$. Let $\mathbf{w} = (w_1, w_2, ..., w_N)'$ be a vector of fixed weights such that $\Sigma_{i=1}^{N} w_i = 1$. Then the average impulse response (or forecast error variance decomposition) of the shock to variable j, at horizon n, is computed as:

$$X_{\omega,j,n} = \sum_{i=1}^{N} w_i X_{i,j,n}. \tag{S12}$$

and its dispersion is computed by:

$$\sigma_{X_{\omega,j,n}} = \left[\sum_{i=1}^{N} w_i^2 \left(X_{i,j,n} - X_{\omega,j,n} \right)^2 \right]^{1/2}, \tag{S13}$$

assuming country-specific impulse responses or forecast error variance decompositions are approximately uncorrelated.

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