International Risk Sharing and Wealth Allocation with Higher Order Cumulants

Online Appendix

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A Summary of the three-country model

In this section we present the model in a way that lends itself for recursive solution, as implemented in the Fortran codes (see `core_model.f90`). In particular, the solution strategy consists of guessing B's and C's prices in country A's consumption units, recursively solving for the other variables and thus verifying the guess. Hence we guess $p_{B,t}$ and $p_{C,t}$. Using the aggregate price equations obtain

$$p_{A,t} = \left(\frac{1 - (1 - \nu_A)\big(\varsigma_A p_{B,t}^{1-\theta} + (1 - \varsigma_A) p_{C,t}^{1-\theta}\big)}{\nu_A} \right)^{\frac{1}{1-\theta}} \tag{A.1}$$

$$Q_{A,B,t} = \left(\nu_B p_{B,t}^{1-\theta} + (1 - \nu_B)\left(\varsigma_B p_{A,t}^{1-\theta} + (1 - \varsigma_B) p_{C,t}^{1-\theta}\right)\right)^{\frac{1}{1-\theta}} \tag{A.2}$$

$$Q_{A,C,t} = \left(\nu_C p_{C,t}^{1-\theta} + (1 - \nu_C)\left(\varsigma_C p_{A,t}^{1-\theta} + (1 - \varsigma_C) p_{B,t}^{1-\theta}\right)\right)^{\frac{1}{1-\theta}} \tag{A.3}$$

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From countries' demand and resource constraints have

$$c_{A,A,t} = \nu_A \left(p_{A,t}\right)^{-\theta} C_{A,t} \tag{A.4a}$$

$$c_{A,B,t} = \varsigma_A \left(1-\nu_A\right) \left(p_{B,t}\right)^{-\theta} C_{A,t} \tag{A.4b}$$

$$c_{A,C,t} = \left(1-\varsigma_A\right) \left(1-\nu_A\right) \left(p_{C,t}\right)^{-\theta} C_{A,t} \tag{A.4c}$$

$$c_{B,B,t} = \nu_B \left(p_{B,t}\right)^{-\theta} C_{B,t} \tag{A.5a}$$

$$c_{B,A,t} = \varsigma_B \left(1-\nu_B\right) \left(\frac{p_{A,t}}{Q_{A,B,t}}\right)^{-\theta} C_{B,t} \tag{A.5b}$$

$$c_{B,C,t} = \left(1-\varsigma_B\right) \left(1-\nu_B\right) \left(\frac{p_{C,t}}{Q_{A,B,t}}\right)^{-\theta} C_{B,t} \tag{A.5c}$$

$$c_{C,C,t} = \nu_C \left(p_{C,t}\right)^{-\theta} C_{C,t} \tag{A.6a}$$

$$c_{C,A,t} = \varsigma_C \left(1-\nu_C\right) \left(\frac{p_{C,t}}{Q_{A,C,t}}\right)^{-\theta} C_{C,t} \tag{A.6b}$$

$$c_{C,B,t} = \left(1-\varsigma_C\right) \left(1-\nu_C\right) \left(\frac{p_{B,t}}{Q_{A,C,t}}\right)^{-\theta} C_{C,t} \tag{A.6c}$$

Using the risk-sharing constants

$$C_{B,t} = Q_{A,B,t}^{-\frac{1}{\rho}} \kappa_{A,B} C_{A,t} \qquad C_{C,t} = Q_{A,C,t}^{-\frac{1}{\rho}} \kappa_{A,C} C_{A,t}. \tag{A.7}$$

the goods market equilibrium for the three countries are given by

$$n_A p_{A,t} Y_{A,t} = \left(p_{A,t}\right)^{1-\theta} \left(n_A \nu_A + n_B \varsigma_B \left(1-\nu_B\right) Q_{A,B,t}^{\theta-\frac{1}{\rho}} \kappa_{A,B} + n_C \varsigma_C \left(1-\nu_C\right) Q_{A,C,t}^{\theta-\frac{1}{\rho}} \kappa_{A,C}\right) C_{A,t} \tag{A.8}$$

$$n_B p_{B,t} Y_{B,t} = \left(p_{B,t}\right)^{1-\theta} \left(n_B \nu_B Q_{A,B,t}^{\theta-\frac{1}{\rho}} \kappa_{A,B} + n_A \varsigma_A \left(1-\nu_A\right) + n_C \left(1-\varsigma_C\right) \left(1-\nu_C\right) Q_{A,C,t}^{\theta-\frac{1}{\rho}} \kappa_{A,C}\right) C_{A,t} \tag{A.9}$$

$$n_C p_{C,t} Y_{C,t} = \left(p_{C,t}\right)^{1-\theta} \left(n_C \nu_C Q_{A,C,t}^{\theta-\frac{1}{\rho}} \kappa_{A,C} + n_A \left(1-\varsigma_A\right) \left(1-\nu_A\right) + n_B \left(1-\varsigma_B\right) \left(1-\nu_B\right) Q_{A,B,t}^{\theta-\frac{1}{\rho}} \kappa_{A,B}\right) C_{A,t}. \tag{A.10}$$

Summing up these three equations gives

$$C_{A,t} = \mu_{A,t} Y_{W,t} \tag{A.11}$$

where

$$\mu_{A,t} = \left[n_A + n_B Q_{A,B,t}^{1-\frac{1}{\rho}} \kappa_{A,B} + n_C Q_{A,C,t}^{1-\frac{1}{\rho}} \kappa_{A,C} \right]^{-1} \tag{A.12}$$

Note that $\mu_{A,t} > 0$ can be larger or smaller than 1. Likewise,by applying equations (A.7) we have

$$C_{B,t} = \mu_{B,t} Y_{W,t}, \text{ and } C_{C,t} = \mu_{C,t} Y_{W,t} \tag{A.13}$$

where per equations (A.7)

$$\mu_{B,t} = \mu_{A,t} \kappa_{A,B} Q_{A,B,t}^{-\frac{1}{\rho}}, \text{ and } \mu_{C,t} = \mu_{A,t} \kappa_{A,C} Q_{A,C,t}^{-\frac{1}{\rho}}. \tag{A.14}$$

Note also that $n_A \mu_A + n_B \mu_B Q_{A,B,t} + n_C \mu_C Q_{A,C,t} = 1$.

Then use the labor supply equations to generate global output, i.e. from

$$L_{A,t} = \left(C_{A,t}^{-\rho} \frac{1-\alpha}{\chi} p_{A,t} D_{A,t} \right)^{\frac{1}{\varphi+\alpha}} \tag{A.15}$$

$$L_{B,t} = \left({C_{B,t}^*}^{-\rho} \frac{1-\alpha}{\chi} p_{B,t} Q_{A,B,t}^{-1} D_{B,t} \right)^{\frac{1}{\varphi+\alpha}} \tag{A.16}$$

$$L_{C,t} = \left({C_{C,t}^*}^{-\rho} \frac{1-\alpha}{\chi} p_{C,t} Q_{A,C,t}^{-1} D_{C,t} \right)^{\frac{1}{\varphi+\alpha}} \tag{A.17}$$

Or, using the results so far

$$n_A p_{A,t} D_{A,t} L_{A,t}^{1-\alpha} = n_A p_{A,t} D_{A,t} \left(\mu_A^{-\rho} \frac{1-\alpha}{\chi} p_{A,t} D_{A,t} \right)^{\frac{(1-\alpha)}{\varphi+\alpha}} Y_{W,t}^{-\frac{\rho(1-\alpha)}{\varphi+\alpha}} \tag{A.18}$$

$$n_B p_{B,t} D_{B,t} L_{B,t}^{1-\alpha} = n_B p_{B,t} D_{B,t} \left(\mu_B^{-\rho} \frac{1-\alpha}{\chi} p_{B,t} Q_{A,B,t}^{-1} D_{B,t} \right)^{\frac{(1-\alpha)}{\varphi+\alpha}} Y_{W,t}^{-\frac{\rho(1-\alpha)}{\varphi+\alpha}} \tag{A.19}$$

$$n_C p_{C,t} D_{C,t} L_{C,t}^{1-\alpha} = n_C p_{C,t} D_{C,t} \left(\mu_C^{-\rho} \frac{1-\alpha}{\chi} p_{C,t} Q_{A,C,t}^{-1} D_{C,t} \right)^{\frac{(1-\alpha)}{\varphi+\alpha}} Y_{W,t}^{-\frac{\rho(1-\alpha)}{\varphi+\alpha}} \tag{A.20}$$

Adding them up yields

$$Y_{W,t} = \begin{bmatrix} n_A p_{A,t} D_{A,t} \left(\mu_A^{-\rho} \frac{1-\alpha}{\chi} p_{A,t} D_{A,t} \right)^{\frac{(1-\alpha)}{\varphi+\alpha}} + \\ n_B p_{B,t} D_{B,t} \left(\mu_B^{-\rho} \frac{1-\alpha}{\chi} p_{B,t} Q_{A,B,t}^{-1} D_{B,t} \right)^{\frac{(1-\alpha)}{\varphi+\alpha}} + \\ n_C p_{C,t} D_{C,t} \left(\mu_C^{-\rho} \frac{1-\alpha}{\chi} p_{C,t} Q_{A,C,t}^{-1} D_{C,t} \right)^{\frac{(1-\alpha)}{\varphi+\alpha}} \end{bmatrix}^{\frac{\varphi+\alpha}{\rho(1-\alpha)+\varphi+\alpha}} \tag{A.21}$$

Finally use

$$n_B Y_{B,t} = (p_{B,t})^{-\theta} \left(n_B \nu_B Q_{A,B,t}^{\theta-\frac{1}{\rho}} \kappa_{A,B} + n_A \varsigma_A (1-\nu_A) + n_C (1-\varsigma_C)(1-\nu_C) Q_{A,C,t}^{\theta-\frac{1}{\rho}} \kappa_{A,C} \right) C_{A,t} \tag{A.22}$$

$$n_C Y_{C,t} = (p_{C,t})^{-\theta} \left(n_C \nu_C Q_{A,C,t}^{\theta-\frac{1}{\rho}} \kappa_{A,C} + n_A (1-\varsigma_A)(1-\nu_A) + n_B (1-\varsigma_B)(1-\nu_B) Q_{A,B,t}^{\theta-\frac{1}{\rho}} \kappa_{A,B} \right) C_{A,t} \tag{A.23}$$

to verify the guess for $p_{B,t}$ and $p_{C,t}$.

A.1 Financial autarky: three-country model

Under autarky each country must consume in each period all the equivalent value of domestic production, i.e. there is only trade in goods. We solve the equilibrium by guessing and verifying $p_{B,t}$ and $p_{C,t}$. We solve using the three price indexes, three autarky resource constraints, three labor market clearing conditions and two goods market clearing condition. Given the guess of $p_{F,t}$ and using aggregate price equations obtain

$$p_{A,t} = \left(\frac{1 - (1-\nu_A)\big(\varsigma_A p_{B,t}^{1-\theta} + (1-\varsigma_A) {p_{C,t}}^{1-\theta}\big)}{\nu_A} \right)^{\frac{1}{1-\theta}} \tag{A.24}$$

$$Q_{A,B,t} = \big(\nu_B p_{B,t}^{1-\theta} + (1-\nu_B)\left(\varsigma_B p_{A,t}^{1-\theta} + (1-\varsigma_B) {p_{C,t}}^{1-\theta}\right)\big)^{\frac{1}{1-\theta}} \tag{A.25}$$

$$Q_{A,C,t} = \big(\nu_C p_{C,t}^{1-\theta} + (1-\nu_C)\left(\varsigma_C p_{A,t}^{1-\theta} + (1-\varsigma_C) {p_{B,t}}^{1-\theta}\right)\big)^{\frac{1}{1-\theta}} \tag{A.26}$$

We solve for consumption by using the budget constraints, together with the labor market equilibrium conditions. Consider the labor market equilibrium conditions, raised to the power of $1-\alpha$ and multiplied by TFP and domestic price. This yields

$$C_{j,t} = p^*_{j,t} D_{j,t} L^{1-\alpha}_{j,t} = Q^{-1}_{A,j,t} p_{j,t} D_{j,t} \left(C^{-\rho}_{j,t} \frac{1-\alpha}{\chi} p_{j,t} Q^{-1}_{A,j,t} D_{j,t} \right)^{\frac{1-\alpha}{\varphi+\alpha}} \tag{A.27}$$

and hence

$$C_{j,t} = \left(Q^{-1}_{A,j,t} p_{j,t} D_{j,t} \left(\frac{1-\alpha}{\chi} p_{j,t} Q^{-1}_{A,j,t} D_{j,t} \right)^{\frac{1-\alpha}{\varphi+\alpha}} \right)^{\frac{\varphi+\alpha}{(\varphi+\alpha)+\rho(1-\alpha)}} \tag{A.28}$$

for country $j = \{A, B, C\}$. The guesses will be correct if these goods-market equilibrium conditions are satisfied:

$$n_B Y_{B,t} = (p_{B,t})^{-\theta} \left(n_B \nu_B Q^{\theta}_{A,B,t} C_{B,t} + n_A \varsigma_A (1-\nu_A) C_{A,t} + n_C (1-\varsigma_C)(1-\nu_C) Q^{\theta}_{A,C,t} C_{C,t} \right) \tag{A.29}$$

$$n_C Y_{C,t} = (p_{C,t})^{-\theta} \left(n_C \nu_C Q^{\theta}_{A,C,t} C_{C,t} + n_A (1-\varsigma_A)(1-\nu_A) C_{A,t} + n_B (1-\varsigma_B)(1-\nu_B) Q^{\theta}_{A,B,t} C_{B,t} \right). \tag{A.30}$$

B Perturbation Methods

B.1 Series expansion and cumulant-generating function

The characterization of the series expansion in the main text is analogous to that used in the macro-finance literature by e.g. Backus et al. (2011). In that case they show that the moment-generating function for a random variable x is (if it exists)

$$h(\omega, x) = E(e^{\omega x}) \tag{B.1}$$

then the cumulant-generating function is simply

$$f(\omega, x) = \ln h(\omega, x) \tag{B.2}$$

Then, assuming that $h(\omega, x)$ is analytical, we can write its series expansion as

$$f(\omega, x) = \sum_{j=1}^{\infty} f_j(x) \frac{\omega^j}{j!} \tag{B.3}$$

where $f_j := \left. \frac{\partial^j \log f(\omega, x)}{\partial \omega^j} \right|_{\omega=0}$.

B.2 A useful efficient procedure

We detail a practical solution step which might be particularly useful for numerical solutions of DSGE models of any size, e.g. using Dynare (Juillard, 1996).[1] We then offer higher-order accurate solution of the Negishi weights. For this illustration we focus on the two-country version of the model, *wlog*.

We illustrate the procedure focusing on Negishi weights, which also happen to feature differently than the other variables in the model.[2] We begin by observing that the risk-sharing condition can be solved backward to yield:

$$\log \zeta_{A,t} - \log \zeta_{B,t} + \log Q_{A,B,t} = \log \zeta_{A,0} - \log \zeta_{B,0} + \log Q_{A,B,0} := \rho \log \kappa_{A,B}. \tag{B.4}$$

where the subscript 0 indicates the time zero in which the risk-sharing agreement is entered for the first time.

The initial distribution of wealth under a full set of period-by-period state contingent (Arrow) securities is pinned down by a condition on the initial distribution of Arrow securities across borders. Take the $m - order$ series-expansion of $\kappa_{A,B}$ around $\omega = 0$

$$\log \kappa_{A,B}(\omega) \approx \kappa^{(0)}_{A,B} + \kappa^{(1)}_{A,B}\omega + \ldots + \kappa^{(m)}_{A,B} \frac{1}{m!}\omega^m \tag{B.5}$$

[1]We solve our model using specific code in Wolfram's Mathematica for which we don't need to use this practical suggestion.

[2]For non-separable preferences or recursive preferences, e.g. à la Bansal and Yaron (2004), a similar decomposition can be obtained. Details are available from the authors on request.

where $\kappa_{A,B}^{(m)} := \left.\frac{\partial^m \log \kappa_{A,B}}{\partial \omega^m}\right|_{\omega=0}$. At each order m, solve for $\kappa_{A,B}^{(m)}$ and proceed recursively, starting from $\kappa_{A,B}^{(0)} = \bar{\kappa}_{A,B}$ and $\kappa_{A,B}^{(1)} = 0$ (as certainty equivalence holds at first order). By way of example, a second order expansion implies that

$$\widetilde{\kappa}_{A,B} := \log \kappa_{A,B}(\omega) - \log \kappa_{A,B}^{(0)} \approx \frac{1}{2}\kappa_{A,B}^{(2)} \tag{B.6}$$

where *wlog* we set $\omega = 1$.[3]

Our solution algorithm (whether applied analytically or numerically) proceeds as follows

1. Expand to the order of interest the system of equations constituting the model;
2. Find the RE solution for all variables as a function of $\widetilde{\kappa}_{A,B}$;
3. Use the appropriate series expansion of budget constraint under the condition $S_{A,0} = 0$ to solve for $\widetilde{\kappa}_{A,B}$.

For higher orders of approximation this algorithm can be used recursively starting from lower orders to build the solution for higher orders, i.e. to construct a solution for each of the variables of the model with the same structure as in equation (B.5).

It is worth stressing that this solution technique does not amount to finding the "risky steady state" à la Coeurdacier et al. (2011). Despite some similarities in the two methods, our technique is specific to the derivation of $\kappa_{A,B}$. It relies on the existence of an explicit condition that can be imposed to solve for $\kappa_{A,B}$—a given initial distribution of assets. The intuition is simple. Under complete markets we can solve the model economy (i.e. find the state-space representation of the endogenous dynamic variables) conditionally on the initial, time-invariant distribution of assets. Consider the

[3] The accuracy of the approximation clearly depends on the size of ω. Nevertheless, we can normalize this to 1 and scale appropriately the standard deviation of the underlying shocks, *wlog*.

representation of a DSGE model under a second order perturbation[4]

$$AE_t\widetilde{X}^{(2)}_{t+1} + B\widetilde{X}^{(2)}_t + CE_t\left(\widetilde{X}^{(1)}_{t+1}\otimes\widetilde{X}^{(1)}_{t+1}\right) + D\left(\widetilde{X}^{(1)}_t\otimes\widetilde{X}^{(1)}_t\right) + F\left(\widetilde{X}^{(1)}_t\otimes\varepsilon_t\right) + G\widetilde{\kappa}_{A,B} = 0 \tag{B.7}$$

where X_t is a vector of variables, $\tilde{X}^{(i)} := X^{(i)} - X^{(0)}$, ε_t is an i.i.d. vector of innovations, A, B, C, D, F, G and H (below) are conformable matrices of coefficients. Importantly, $\kappa_{A,B}$ does not appear in the coefficient matrices A, B, C, D, F, G which are only reflecting deterministic steady state information. As argued above, the non-linear terms are of lower order (here first order). In this example these terms are solved separately from the system

$$E_tA\widetilde{X}^{(1)}_{t+1} + B\widetilde{X}^{(1)}_t + H\varepsilon_t = 0. \tag{B.8}$$

Notably, $\widetilde{\kappa}^{(1)}_{A,B}$ is missing from the first order, as it is zero under certainty equivalence.

Since $\widetilde{\kappa}^{(2)}_{A,B}$ is time invariant, we can re-write equation (B.7) as

$$AE_t\check{X}^{(2)}_{t+1} + B\check{X}^{(2)}_t + CE_t\left(\widetilde{X}^{(1)}_{t+1}\otimes\widetilde{X}^{(1)}_{t+1}\right) + D\left(\widetilde{X}^{(1)}_t\otimes\widetilde{X}^{(1)}_t\right) + F\left(\widetilde{X}^{(1)}_t\otimes\varepsilon_t\right) = 0 \tag{B.9}$$

where $\check{X}^{(2)}_t = \widetilde{X}^{(2)}_t + (A+B)^{-1}G\widetilde{\kappa}^{(2)}_{A,B}$. Note that under complete markets the system (B.9) does not need to include the households budget constraint, which will instead be used in a second step to solve for $\widetilde{\kappa}^{(2)}_{A,B}$.

Summing up, in the class of models where $\kappa_{A,B}$ enters log linearly,[5] the solution procedure consists naturally of two steps: i) solve for allocations and prices using system (B.9); ii) use the (approximated) budget constraint at time 0 to solve for $\widetilde{\kappa}^{(2)}_{A,B}$. These two steps will then allow to recover $\widetilde{X}^{(2)}_t$. The same procedure holds for any order of approximation.

To avoid misinterpretations of the algorithm, it is important to stress that there

[4]To simplify the illustration we assume that the model has at most one period ahead expectations and that that the system is represented in "companion form" with all lags subsumed in the vector X_t.

[5]This is the case of not only CRRA preferences, but also Epstein-Zin preferences.

is only one (standard) perturbation taking place, i.e. along the "risk" loading parameter ω. The solution of the resulting system of series expansions is recursive with respect to $\kappa_{A,B}$ at each order of approximation – at least in this class of models. The solution could as well be obtained all at once (less efficiently). The key is that we go from the original non-linear model to a system of series expansions of all the "risk-sensitive" variables, including $\kappa_{A,B}$.[6]

B.2.1 Higher-order accurate solution of the Negishi weights

In the simple Lucas' endowment model discussed in the main text, the risk-sharing constant $\kappa_{A,B}$,[7] defined as $\kappa_{A,B} = \dfrac{C_{B,t}}{C_{A,t}}$ depends recursively on exogenous variables, i.e.

$$\frac{1}{n + (1-n)\kappa_{A,B}} = \frac{E_0 \sum_{t=0}^{\infty} \delta^t \left(D_{w,t}\right)^{-\rho} D_{A,t}}{E_0 \sum_{t=0}^{\infty} \delta^t \left(D_{w,t}\right)^{-\rho} D_{w,t}} \tag{B.10}$$

where $D_{w,t} = n_A D_{A,t} + n_B D_{B,t}$.

In general, and specifically when there is production, the right-hand-side of equation (B.10) is endogenous (e.g. the discount factor depends on consumption, and income depends on production). Therefore, to find the risk sharing constant in this more general setting we typically need to use some fixed-point algorithm.

We propose a closed-form solution for $\kappa_{A,B}$ based on perturbation methods. We present a version that is accurate up to second order but that is straightforward to extend to higher orders. It is important to reiterate that perturbation-based solutions of $\kappa_{A,B}$ don't require any ad-hoc deviation from standard perturbation principles.[8] Furthermore, the recursivity of the solution that we propose is a natural property of models

[6] A similar recursivity between dynamic allocations and time-invariant financial allocations emerges in long-run portfolio decisions, as discussed by Devereux and Sutherland (2011). There, the "zero order" asymptotic portfolio composition can be solved recursively from the dynamic allocation of the model's variables. This is possible since the "zero order" portfolio shares are time-invariant (like our $\kappa_{A,B}$) and the dynamics of the model can be defined conditionally on these shares. Like for the portfolio case, no ad-hoc assumptions or heuristic techniques are needed. The solution emerges from the mechanical application of series expansion techniques and solution techniques for dynamic rational expectation models.

[7] The term "constant" refers to time invariance. This coefficient is not invariant to risk.

[8] The key principle is that to solve at higher orders of accuracy, powers of the variables in the series expansion must be computed with the solution of a lower order of accuracy Holmes (e.g. 1995).

with complete markets. Specifically, while the dynamic equilibrium can be determined conditionally on arbitrary values for $\kappa_{A,B}$, the latter can be determined using an equation that is redundant for the dynamic equilibrium under complete markets, i.e. the law of motion of Arrow-Debreu securities.

There are possibly various strategies to compute $\kappa_{A,B}$. For this paper we implement two. One, computed using Wolfram Mathematica, literally represents the solution of the endogenous variables as functions of $\kappa_{A,B}$ and then uses these solutions in the law of motion of Arrow-Debreu securities to solve for $\kappa_{A,B}$. The other approach is more suited for numerical solutions of DSGE models, e.g. using Dynare.

We can describe this second approach as follows. The solution is based on the observation that, up to second order of accuracy, $\kappa_{A,B}$ depends on second order terms, and in particular on the variance of the exogenous processes (a similar logic applies to higher orders). We can thus represent the Negishi weights as

$$\log(\kappa_{A,B}) := \bar{\kappa}_{A,B} E_t \varepsilon^2_{\kappa,t+1} \tag{B.11}$$

so that

$$\bar{\kappa}_{A,B} E_t \varepsilon^2_{\kappa,t+1} = \widetilde{C}_{B,t} - \widetilde{C}_{A,t} \tag{B.12}$$

where $\bar{\kappa}_{A,B}$ is the unknown parameter we want to solve for, and $\varepsilon_{\kappa,t}$, is a mean-zero iid auxiliary shock with variance denoted by σ^2_κ. This implies that $E_t \varepsilon^2_{\kappa,t+1} = \sigma^2_\kappa$. So far we have thus re-scaled the original kappa by σ^2_κ.

The second-order solution of a DSGE model can be written in a second-order VAR form as (e.g. following Dynare notation)

$$Y_{A,t} = A y_{t-1} + B u_t + \frac{1}{2} \left[C \left(y_{t-1} \otimes y_{t-1} \right) + D \left(u_t \otimes u_t \right) + 2F \left(y_{t-1} \otimes u_t \right) \right] + \frac{1}{2} G \vec{\Sigma^2} \tag{B.13}$$

where $Y_{A,t} \in \mathbb{R}^{n_y}$ is the vector of all the n_y variables (endogenous and exogenous excluding innovations), $u_t \in \mathbb{R}^{n_i}$ is the vector of all the n_i (iid) innovations, A, B, C, D, F, G are conformable matrices, and for any column vectors x and z, $(x \otimes z)$ is the vectorized outer product of these vectors. $\Sigma^2 := E_t\left(u_{t+1}u'_{t+1}\right)$, and $\vec{\cdot}$ is the vectorization operator.[9]

The key term in equation (B.13) is the last one, which shifts the mean of variables in proportion to the exogenous risk, captured by the variance matrix Σ^2 (also referred to as the stochastic steady state in the literature). Using regular perturbations (see e.g. Lombardo and Uhlig, 2018), none of the matrices in (B.13) depends on exogenous risk. This means that the only place where σ_κ appears is in Σ^2.

The vector $Y_{A,t}$ contains the variable measuring Arrow-Debreu securities. Assume the latter are in position i_{AD}, and that σ_κ occupies position j_{σ_κ} in the vector $\vec{\Sigma^2}$. Then we have that

$$\begin{aligned} Y_{A,t}[i_{AD}] &= A[i_{AD},:]y_{t-1} + B[i_{AD},:]u_t + \frac{1}{2}\left[C[i_{AD},:]\left(y_{t-1}\otimes y_{t-1}\right)\right.\\ &\left.+D[i_{AD},:]\left(u_t\otimes u_t\right) + 2F[i_{AD},:]\left(y_{t-1}\otimes u_t\right)\right] + \frac{1}{2}G[i_{AD},:]\vec{\Sigma^2} \end{aligned} \tag{B.14}$$

where for a matrix X, $X[i,j]$ denotes the element in row i and column j, and where $X[i,:]$ denotes the row i of matrix X; for a vector z, $z[j_{\sigma_\kappa}]$ is the $j_{\sigma_\kappa} - th$ element in z. In particular, $\vec{\Sigma^2}[j_{\sigma_\kappa}] = \sigma_\kappa^2$.

Note that if we set $\bar{\kappa}_{A,B} = 1$, we can solve for σ_κ^2 that satisfies some restriction on $Y_{A,t}[i_{AD}]$. In particular we know that under complete markets it must be that $y_0[i_{AD}] = 0$ (Ljungqvist and Sargent, 2012). One way to implement this condition is to assume that at time 0 and -1 the economy was at the stochastic steady state, i.e. all elements of equation (B.14) are zero except the last one, i.e.[10]

$$y_0[i_{AD}] = 0 = \frac{1}{2}G[i_{AD},:]\vec{\Sigma^2} \tag{B.15}$$

[9]To date, Dynare returns only the product $G\vec{\Sigma^2}$ in the variable "oo_.dr.ghs2". In order to implement our algorithm this product must be factorized in the two components. This can be easily done by modifying Dynare function `dyn_second_order_solver.m` at about line 173, by adding a new variable e.g. `dr.G=LHS\(-RHS);`, where LHS and RHS are variables defined in the function.

[10]Equally easily implementable is any other condition, e.g. $Ey_0[i_{AD}] = 0$.

Then we can solve for σ_κ^2 as

$$\sigma_\kappa^2 = -\frac{G[i_{AD}, j_{\sigma_\kappa}^{\perp}]\vec{\Sigma}^2[j_{\sigma_\kappa}^{\perp}]}{G[i_{AD}, j_{\sigma_\kappa}]} \tag{B.16}$$

where $j_{\sigma_\kappa}^{\perp}$ denotes all the elements excluding j_{σ_κ}. Now we simply need to swap values, i.e. $\bar{\kappa}_{A,B} \leftarrow \sigma_\kappa^2$ $\sigma_\kappa^2 \leftarrow 1$. With this assignment of values, $\kappa_{A,B}$ is the second-order accurate risk-sharing constant that implements complete markets.

Our proposed algorithm, correctly implements complete markets up to second order accuracy. It should be noted also that our approach does not affect the first-order solution. This solution correctly describes growth rates of variables, since the risk-sharing constant is invariant to time (Ljungqvist and Sargent, 2012).

This approach is reminiscent of the solution algorithm proposed by Devereux and Sutherland (2011) (DS) to solve for portfolio shares up to second order. DS introduce an auxiliary iid shock in the budget constraint of investors as a placeholder for portfolio shares. By knowing the position of this auxiliary shock DS can then use simple linear algebra to derive the shares. Although we solve a different problem, our algorithm shares with DS the idea of using auxiliary iid shocks as placeholders for parameters that would otherwise drop out of the perturbed solution.

C Analytical results

In the text, we have referred to $a_\bullet$ and $b_\bullet$ as complex convolutions of deep parameters (ρ, θ, δ and n_A). Below we show the analytical expression for $a_\bullet$ and $b_\bullet$ for the special case of $n_A = \frac{1}{2}$:

$$a_{\kappa,2} = \frac{1}{4(1-\delta)} \tag{C.1}$$

$$a_{\kappa,2^2} = \frac{\rho}{32(1-\delta)} \tag{C.2}$$

$$a_{\kappa,3} = \frac{1}{12(1-\delta)} \tag{C.3}$$

$$a_{\kappa,4} = \frac{1}{48(1-\delta)} \tag{C.4}$$

$$a_{\kappa,\Gamma} = \frac{\delta(\rho-1)(\theta(\rho-2)-1)}{32(1-\delta)\theta} \tag{C.5}$$

$$a_{\gamma,A} = \frac{\delta(2\theta-2\theta^2+\rho-4\theta\rho+3\theta^2\rho)}{8(1-\delta)\theta^2} \tag{C.6}$$

$$a_{\gamma,B} = \frac{\beta(\rho-\theta^2-2\theta)}{8(1-\delta)\theta^2} \tag{C.7}$$

$$a_{\phi,A} = \frac{\delta(\theta-1)(\rho-1)(-2-\rho+\theta(1+5\rho)-\theta^2(7\rho-4))}{48(1-\delta)\theta^3} \tag{C.8}$$

$$a_{\phi,B} = \frac{\delta(\theta-1)(\rho-1)(-2-\rho-\theta(-1+\rho)-\rho-\theta^2(\rho-4))}{48(1-\delta)\theta^3} \tag{C.9}$$

$$a_{\eta,A} = \frac{\delta(\theta-1)}{384(1-\delta)\theta^4}(\rho(-6+3\rho+\rho^2)-\theta^2(-4+27\rho^2-17\rho^3)- \tag{C.10}$$

$$\theta(8-24\rho+3\rho^2+7\rho^3)-\theta^3(-8+30\rho-39\rho^2+15\rho^3)) \tag{C.11}$$

$$a_{\eta,B} = \frac{\delta(\theta-1)}{384(1-\delta)\theta^4}(\rho(-6+3\rho+\rho^2)-\theta^2(-4+3\rho^2-\rho^3)- \tag{C.12}$$

$$\theta(8-3\rho+\rho^2)-\theta^3(-8+18\rho-9\rho^2+\rho^3)) \tag{C.13}$$

$$b_\gamma = \frac{\delta(\theta-1)(\rho-1)}{\theta} \tag{C.14}$$

$$b_\phi = \frac{\delta(\theta-1)(2-\theta-3\theta\rho-\theta^2(4-9\rho+3\rho^2)}{4\theta^3} \tag{C.15}$$

$$b_{\gamma_A^2} = \frac{\delta(\theta-1)(\rho-1)(-3\beta\theta(1+\theta(\rho-2))(\rho-1)}{2\theta^3} \tag{C.16}$$

$$b_{\gamma_B^2} = -\frac{\delta(\theta-1)(\rho-1)(-3\beta\theta(1+\theta(\rho-2))(\rho-1)}{2\theta^3} \tag{C.17}$$

$$b_{\gamma_B\gamma_A} = 0 \tag{C.18}$$

$$b_\eta = \frac{\delta(\theta-1)(\rho-1)(-2+\theta+3\theta\rho+\theta^2(2-5\rho+\rho^2)}{2\theta^3} \tag{C.19}$$

Table 1: Values of parameters a and b under our calibration and $n_A = 0.5$

$a_{\kappa,2}$	$a_{\kappa,2^2}$	$a_{\kappa,3}$	$a_{\kappa,4}$	$a_{\kappa,\Gamma}$	$a_{\gamma,A}$	$a_{\gamma,B}$	$a_{\phi,A}$
12.5	6.25	4.17	1.04	12.25	14.97	9.52	12.93

$a_{\phi,B}$	$a_{\eta,A}$	$a_{\eta,B}$	b_γ	b_ϕ	$b_{\gamma_A^2}$	$b_{\gamma_B^2}$	$b_{\gamma_A\gamma_B}$	$b_{\eta,B}$
4.73	6.94	1.22	0.98	1.94	11.52	11.52	0	2.83

D Generating skewed-leptokurtic distributions

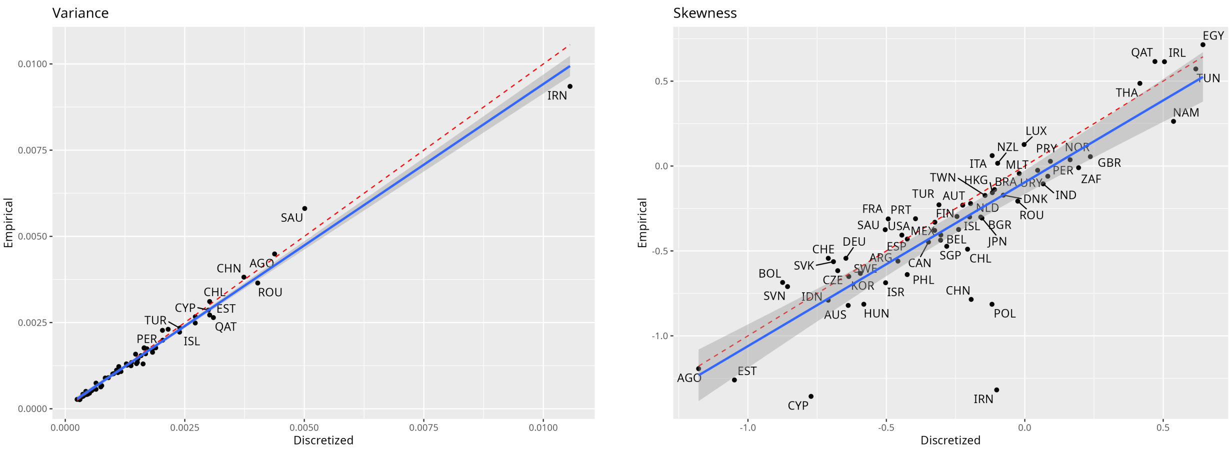
We generate skewed and leptokurtic distributions by adopting the mixed-Gaussian distribution approach discussed Farmer and Toda (2017, FT henceforth).[11] In particular we proceed in two steps. First, we calibrate the parameters of a three-elements Gaussian mixture (i.e. weights, means and standard deviations of each element) by minimizing the relative distance between the observed moments and those generated by this distribution. Second, we implement the algorithm suggested by FT. This consists of matching low-order moments of the conditional distribution using relative entropy as the objective function (i.e. the Kullback-Leibler information).[12]

From the PWT version 10.01, we take the distribution of detrended log real GDP (national accounts measure) as a proxy of country risk (in our model as proxy for the TFP distribution). We remove countries that display very irregular GDP series (these typically consist of countries torn by long conflicts, regime changes, extreme poverty or simply very short series). We further drop countries at the lower and upper 5% of the distribution of kurtosis (of the cyclical component; see below). We take kurtosis as trimming criterion as it is the one with most extreme variation among moments. Our final sample consists of 151 countries with annual series ending in 2019 and starting at variable dates.

[11] See also Civale et al. (2017). The empirically relevant case is of leptokurtic distributions (fatter tails than the Gaussian). We thus refer for simplicity to leptokurtosis as shorthand for non-Gaussian kurtosis.

[12] We refer to Farmer and Toda (2017) for details. We implemented their method converting and adapting their Matlab codes into Python.

Figure 1: Correlation between empirical and discretization-based moments

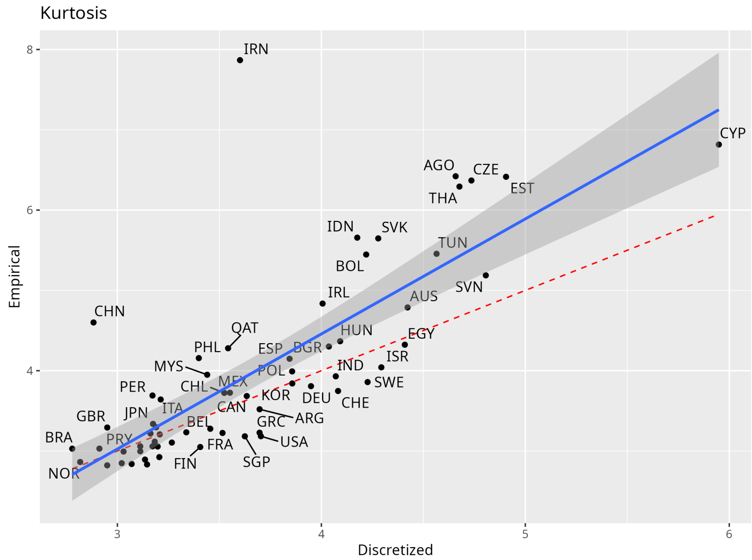


Note: Empirical moments obtained from the simulation of an estimated AR(1) process for the cyclical component of GDP (trend estimated using the Phillips and Shi (2020) method). "Discretized" moments obtained from the simulation of a discretized Gaussian mixture model (à la Farmer and Toda, 2017) calibrated on empirical moments. The solid blue line is the least-square regression. The gray area represents the 95% confidence band. The dashed red line is the 45-degree line.

On the basis of this data we proceed as follows. First, we log-detrend real GDP using the "boosted" HP (Hodrick and Prescott, 1997) filter developed by Phillips and Shi (2020). Second, we fit an AR(1) process on detrended component of log-GDP. Third, we compute the first four moments of the exogenous component of the AR(1) process (the residual). Fourth, we use these moments to calibrate a mixed Gaussian distribution for each country. Fifth, and finally, we discretize the mixed Gaussian using the FT method. Figures 1-2 shows the alignment between the empirical moments (obtained by simulating the estimated AR process) and those obtained by simulating the discretized distribution.[13].

[13]As discussed by FT, the discretization is not always feasible. In particular, FT decrease the number of moments to be matched for each point on the grid of state variables of the AR(1) process, depending on the success of matching the highest moment targeted. While our fourth step yields negligible residuals for all countries, the fifth step (the FT method) generates heterogeneous results across countries: not all tuples of moments can be matched equally well by a Gaussian mixture. To our knowledge this is an unavoidable limitation of matching more than the first two moments of empirical distributions.

Figure 2: Correlation between empirical and discretization-based moments



Note: See Figure 1

E Generation of an artificial sample of economies

A mixed-Gaussian distribution (with the underlying constraints on its parameters) can map into a limited set of values for its moments. Only within this set it is possible to draw moments that are independent of each other. Along the border of this set, changing one moment requires adjusting other moments, thus generating a correlation among them. Algorithm 1 is designed to draw from the set of admissible moments (second to fourth) using a three-term mixed Gaussian distribution.

To search for the admissible set we drew 100,000 values for the second-to-fourth uncentered moments from uniform distributions. The limits of these distributions were set so that: $stdev \in (0.001, .15)$, $skewness \in (-3, 3)$ and $kurtosis \in (3, 20)$. Moreover, we drew the weights of the first two terms of the mixed-Gaussian distribution from the $(0, 1)$ interval (imposing that the sum of all three weights should be 1). Finally the mean of the second and third Gaussian terms were drawn from the $(-1, 1)$ interval; the mean of the first term being set so to obtain a zero-mean for the whole mixed Gaussian distribution.

Algorithm 1 Construction of the artificial sample of countries

1: Express the TFP process of each country in terms of a three-term mixed Gaussian distribution, e.g. for country A (analogously for country B)

$$\sigma_A\varepsilon_{A,t} \approx p_1\mathcal{N}(m_1, v_1) + p_2\mathcal{N}(m_2, v_2) + (1 - p_1 - p_2)\mathcal{N}(m_3, v_3)$$

where $\mathcal{N}(m_i, v_i)$ is the Gaussian PDF with mean m_i and variance v_i and p_i is the weight of the i term in the mixed Gaussian distribution;

2: Express the parameters v_1, v_2 and v_3 in terms of $\gamma_A = E\left(\sigma_A^2\varepsilon_{A,t}^2\right)$, $\phi_A = E\left(\sigma_A^3\varepsilon_{A,t}^3\right)$ and $\eta_A = E\left(\sigma_A^4\varepsilon_{A,t}^4\right)$ and the other parameters of the distributions.

3: Draw random values (from uniform distributions) for p_1, p_2,γ_A, ϕ_A, η_A, m_2 and m_3. Where m_1 is pinned down by the assumption that $E(\sigma_A\varepsilon_{A,t}) = 0$;

4: Discretize the resulting mixed-Gaussian distributions. Denote by N_D the number of these distributions;

5: Use the N_D distributions to parameterize the TFP process of country A in N_D economies, where sizes are set randomly and country B is the same in all of the N_D economies.

6: Find the solution for all the N_D economies.

Figure 3 shows a sub-set of variables of the artificial sample generated by following Algorithm 1. Particularly noteworthy is are the scatter plots relating the three moments of interest. For example, the third and fourth columns of the second-last row show the set of admissible values for the fourth uncentered moment mapped against the second and third moments. Only within the set it is possible to pick independent moments.

F Comparison of the perturbation solution with the global solution

In the main text we have argued that the perturbation-based solution of the model delivers qualitatively reliable results. This section offers an example of the gap between the perturbation and global solution for a key variable in our analysis, κ. For this purpose we solve the two-country model calibrated using the 477 ad-hoc constellations of moments and sizes as well as the baseline parametrization discussed in the text. In the perturbation method we use the ergodic moments implied by the discretization of the DGP for home

Figure 3: Distributions of the key variables in our ad-hoc sample.

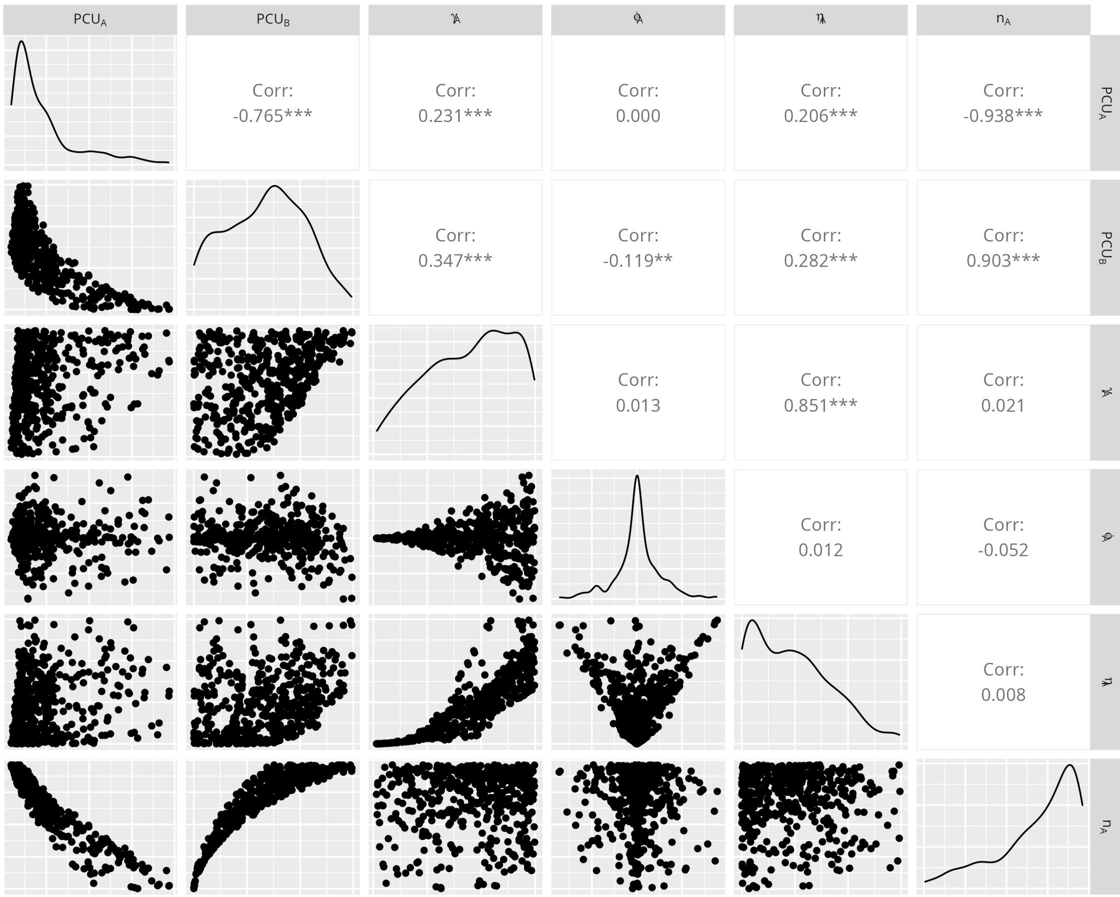
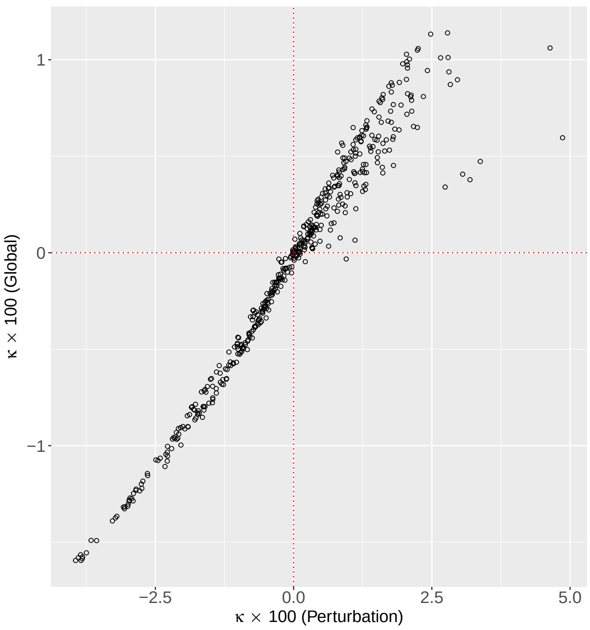


Figure 4: Comparison of solution methods for κ



and foreign TFP.

Figure 4 show the value of $\kappa_{A,B}$ (κ for short) obtained using perturbation method (x-axis) against the value obtained using global methods (y-axis). The two methods give mostly consistent results concerning the sign of (the log of) κ (the correlation is 0.97). That said, perturbation methods tend to overstate the magnitude of κ. Although using the ad-hoc sample magnifies the discrepancies (as it include more diverse values for the moments), these result warrant the use of global solution methods.

G Unpacking the RoW into smaller units

To gauge whether subdividing the large regions into smaller units will affect our results, note that by the equilibrium expression for $\mu_{A,0}$

$$\mu_{A,0} = \left[n_A + (1 - n_A) \sum_{i=1}^{N} N^{-1} Q_{A,i,0}^{\frac{\rho-1}{\rho}} \kappa_{A,i} \right]^{-1} \tag{G.1}$$

a country consumption share of total output will not vary as we increase N only if $Q_{A,i,0}^{\frac{\rho-1}{\rho}}\kappa_{A,i}$ remains constant. Consumption smoothing should not be affected by this sub-partition of RoW, as the SDF will still be determined by global income. But the regression results in the main text suggest that both κ and Q (via ToT)[14] vary with size, albeit considerably less than one-to-one. We can thus conclude that breaking up RoW into sub-units does have material implications for country A's consumption share and welfare. Quantifying this effect would require solving our model for a sufficiently large number of countries, at a very large computational cost.[15]

H Sign switches of the cumulants' effects

In the text, we have argued that the the *sign* of cumulants' (risk's) effect on welfare gains and on relative asset prices depends on the precise numerical constellation of deep parameters: trade elasticity, risk aversion and size. These parameters have a bearing on relative income risk.

Here we show that the predictions of the perturbation solution are remarkably close to true properties of the model. Features that still lack a precise economic intuition.

Among these properties are non-linearities that may arise from skewness. For transparency, here we make the case assuming parameter values that considerably simplify the analysis. In particular we posit $\theta \to \infty$, $n_A = n_B = 0.5$, $\alpha = 1$ and $\beta = 0.98$—so that we are left with one single deep parameter to care about. Moreover, we assume that $D_{B,t} = \bar{D}_B = 1$ and $\varphi_D = 0$—so that the only source of risk comes from country A's *iid* TFP shock—and that $D_{A,0} = \bar{D}_A = 1$. Under these numerical assumptions, the asset

[14]Note that $ToT_{au,0} = 1$.

[15]Solving our three country model starting from a reasonable guesses of the state-space (e.g. solving the equal-size case starting from the heterogeneous-size solution) takes approximately 19 hours on a Laptop with 4-core Intel(R) Core(TM) i7-4810MQ CPU @ 2.80GHz (3.80 max).

price equations reduce to

$$P_{A,K,0} = \frac{\beta}{1-\beta} 0.5^{-\rho} E_0 \left[\left(D_{A,t} + \bar{D}_B \right)^{-\rho} D_{A,t} \right] \tag{H.1a}$$

$$P_{B,K,0} = \frac{\beta}{1-\beta} 0.5^{-\rho} E_0 \left[\left(D_{A,t} + \bar{D}_B \right)^{-\rho} \bar{D}_B \right] \tag{H.1b}$$

where, by our Proposition 1, we know that (holding PPP) the difference in asset prices maps into differences in welfare.

We posit a skewed normal distribution for $D_{A,t}$,[16] and compute $P_{K,A,0} - P_{K,B,0}$ for different degrees of risk aversion (ρ). Figure 5 shows the slope coefficient of regressing 601 realizations of time-0 asset price differences on the corresponding third moment of $D_{A,t}$ (vertical axis) against different values of ρ (horizontal axis). Remarkably, the price difference (hence the difference in welfare under our assumptions) switches sign twice. In line with the result in this figure, by taking a series expansion of equations (H.1a) and (H.1b) we can pin down an approximation of the values of ρ at which the sign switch occurs: the coefficient on the third moment switches sign twice at $\rho = 0.54$ and $\rho = 2.46$.[17]

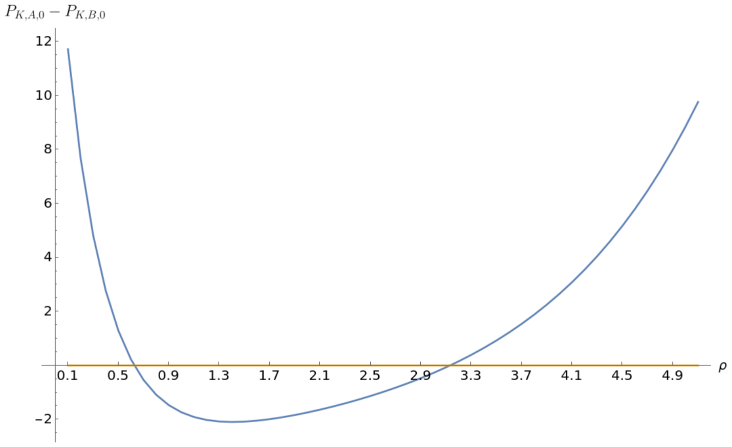
The model predicts that, under perfect risk sharing, the valuation of the assets in the country with the larger negative skewness is lower for ρ's in the range entertained in many quantitative studies, but can, counter-intuitively, be higher for either very low or sufficiently high values of ρ—often advocated to match equity premia.[18] The interest in this result lies in unveiling how, in the model, the combination of LE and SE shaping

[16]The PDF of a variable x with skewed-normal probability distribution is $f(x) = \frac{2}{\sigma}\phi\left(\frac{x-\mu}{\sigma}\right)\Phi\left(\xi\left(\frac{x-\mu}{\sigma}\right)\right)$, where μ is the location parameter, σ is the scale parameter, ξ is the skewness-controlling parameter, $\phi(\cdot)$ is the PDF of the Gaussian distribution and $\Phi(\cdot)$ is its CDF. We can thus adjust μ and σ to keep mean and variance constant while changing ξ to obtain different degrees of skewness. In each simulation we draw $10K$ values for $D_{A,t}$, 801 values for the skewness parameter (from -20 to 20, $step = 0.05$) and 51 values for ρ (from 0.1 to 5.1, $step = 0.1$). The implied range of the third un-centered moment ($E\left(D^3_{A,t}\right)$) is ± 1.8.

[17]While not necessarily quantitatively precise (given the known limits of perturbation methods), the approximation nonetheless provides an accurate *qualitative* prediction that we can verify using our global solution.

[18]For example, in the open-macro literature, the relative risk-aversion parameter (with CRRA preferences) is often 1 (the log case), or slightly above (e.g. Devereux and Engel, 2007 Obstfeld and Rogoff, 2000). In the equity-premium literature, with CRRA preference, values often need to be larger. See the review by Mehra and Prescott (2003) and Cochrane (2008). Higher values are typically inconsistent with estimates of the intertemporal elasticity of substitution (ρ^{-1} under CRRA), calling for preferences that allow for a separation of the two parameters à la Epstein and Zin (1989). We consider these preferences in Appendix I using global solution methods.

Figure 5: Relative asset-price effect of $D_{A,t}$ skewness by varying ρ: sign-switches



the gains from trade in assets may vary with the curvature of the utility function—in our example we focus on one homogeneous goods, hence ruling out income effects from relative output price adjustment. The change is entirely driven by the equilibrium process of the stochastic discount factor.

I Epstein-Zin preferences

It is well known that CRRA preferences generate limited risk premia (and thus gains from risk sharing) for empirically plausible degrees of intertemporal substitution. This is due to the constraint that the degree of risk aversion and the (inverse of the) elasticity of intertemporal substitution are identical with CRRA preferences. Epstein-Zin preferences (Epstein and Zin, 1989) allow to separate these parameters and have been widely used to increase the role of risk in asset prices and welfare. These points have been widely discussed in the literature, e.g. Epstein and Zin (1991), Lewis and Liu (2015), Rudebusch and Swanson (2012), Coeurdacier et al. (2019).

In this appendix we replace the CRRA preferences with Epstein-Zin preferences in our model, and use global methods to assess the gains from risk sharing under asymmetric higher moments and country size.

Epstein-Zin preferences introduce (three) extra state variables under complete markets and thus considerably slow down the solution algorithm. For this reason we only solve the two-country version of our model for four different set of moments and size, calibrated after Germany (DEU), Mexico (MEX), Brazil (BRA) and Thailand (THA).

I.1 Epstein-Zin preferences and Negishi weights

We follow Rudebusch and Swanson (2012) by assuming that the utility kernel of the Epstein-Zin (EZ) preferences, e.g. for country A, takes the form

$$u(C,l) = \frac{C_{A,t}^{1-\rho}}{1-\rho} - \chi \frac{L_{A,t}^{1+\varphi}}{1+\varphi}. \tag{I.1}$$

To avoid that the utility changes sign, these authors suggest to set $\rho > 1$ and define the value function as

$$V_{A,t} = (1-\beta)\left(\frac{C_{A,t}^{1-\rho}}{1-\rho} - \chi \frac{l_{A,t}^{1+\varphi}}{1+\varphi}\right) - \beta \left[E_t\left((-V_{A,t+1})^{\varsigma}\right)\right]^{\frac{1}{\varsigma}} \tag{I.2}$$

where $V_{A,t} < 0$ is the value function, where ς parameterizes risk-aversion. The larger ς the higher is risk aversion. If $\varsigma = 1$ preferences fall back to the CRRA case.

Focusing on the first order conditions related to the consumption and asset choices it can be shown that the risk-sharing condition reduces to

$$-\rho\left(\log C_{A,t} - \log C_{B,t}\right) = \rho \log \kappa + \log Q_{A,B,t} + \Omega_t^S \tag{I.3}$$

where $\rho \log \kappa = \log \lambda_{A,0} - \log \lambda_{B,0} + \log Q_{A,B,0}$ and with $\lambda_{A,t}$ denoting the Lagrange multiplier on the budget constraint of the households (i.e. the marginal utility of consumption), and where

$$\Omega_t^S = \Omega_t^* - \Omega_t + \Omega_{t-1}^S \tag{I.4}$$

such that

$$\Omega_t = \log \beta + \log \left[E_{t-1} \left(V_t^{\varsigma} \right) \right]^{\frac{1}{\varsigma} - 1} + \log \left(V_t^{\varsigma - 1} \right) . \tag{I.5}$$

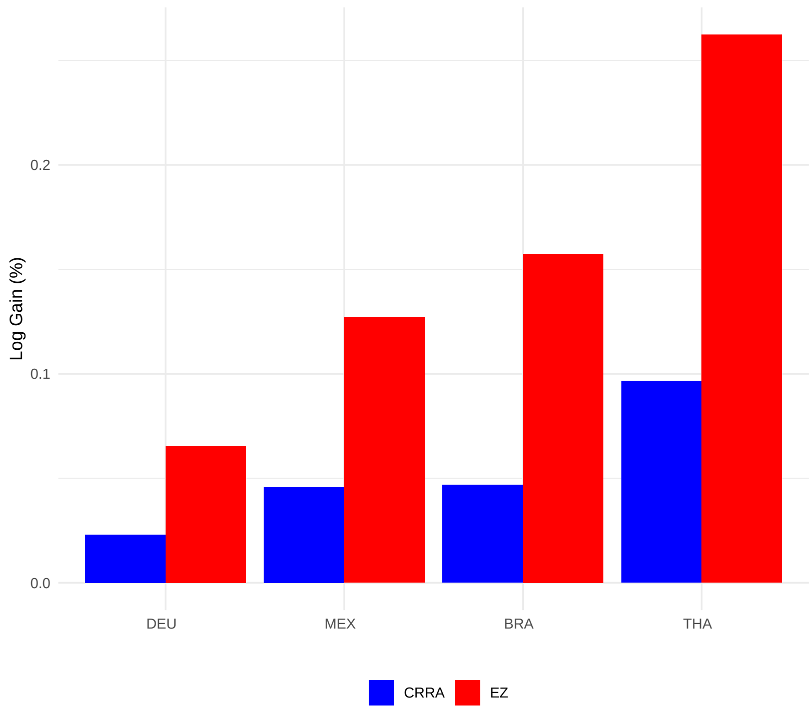
Equations (I.4) and (I.5) imply that the state-space solution of the model under EZ preferences depends on more state variables than the CRRA case. Furthermore, it can be shown that lagged expectations imply that the lagged exogenous shocks (TFP) will appear among the state variables. Hence, in total we have three more state variables relative to the CRRA case in the main text.

The EZ case also implies that the level effect (LE) does not reflect directly the (exchange-rate adjusted) relative consumption. The latter is varying over time, unlike the CRRA case. Our definition of LE, captured by κ, still relates to the time-zero (exchange-rate adjusted) relative marginal utility of consumption, and is thus time invariant.

We solve the model using time-iteration techniques as described in the main text and adapted for the larger set of state variables. Rudebusch and Swanson (2012) suggest very large values for ς. The larger this parameter, though the harder is to find the model solution. This requires very small values of the "learning" (aka diffusion) parameter, and thus longer time to convergence (see Maliar and Maliar, 2014 for details). Furthermore, starting from the CRRA case ($\varsigma = 1$) we increase risk only gradually, taking the solution at the lower value as the new starting point.

Rudebusch and Swanson (2012) show that EZ with preferences as in equation (I.2) the implied consumption-based coefficient of relative risk aversion depends on labor in a complex way. They target a coefficient of 75, which implies a value of ς of 149. We consider an intermediate maximum value of $\varsigma = 30.5$. All the other parameters have the same values as in the baseline model discussed in the text, for both types of preferences. The results are shown in Figure 6, comparing the $\varsigma = 1$ case (i.e. CRRA preferences) with this higher risk-aversion scenario (EZ preferences). Under our conservative calibration of EZ preferences the gains are more than twice those generated under CRRA preferences.

Figure 6: Comparison of gains from risk sharing under CRRA and EZ preferences



J Correlation of TFP across countries and gains from risk sharing

In the main text, for the sake of simplicity, we have assumed that the stochastic processes for TFP are independent across countries. It is will understood that these processes may nonetheless reflect some degree of correlation of the underlying sources of risk. The traditional IRBC literature, e.g. Backus et al. (1992) indeed assumes that the TFP processes are correlated across countries. In this appendix we briefly discuss the implication for risk sharing. We do so analytically focusing on the instructive limit case in which shocks are perfectly correlated, yet they hit countries with different intensities. Without loss of generality, we develop our argument using the the two-country version of the model.

In particular, we assume that

$$D_{B,t} = \zeta D_{A,t};\ \zeta \geq 0. \tag{J.1}$$

Equation (J.1) implies that $corr\left(D_{A,t}, D_{B,t}\right) = 1$. Yet, the intensity of the shocks differs across countries. In a stylized yet compelling way, this case captures one of the core premises of our analysis—strong evidence that shocks, even when global in nature, affect countries in largely asymmetric ways.

To show that perfect correlation of shocks does not rule out gains from risk sharing, we solve the model imposing equation (J.1). Focusing on skewness, up to fourth order of accuracy, we will have:

$$\frac{\partial RGRS}{\partial \phi_A} = \frac{\beta(\zeta-1)^3(\theta-1)^2 n^2 \rho(\theta\rho+\theta+\rho-3)}{2(\beta-1)\theta^3} + \frac{\beta(\zeta-1)^2(\theta-1)^2\rho(\zeta(2\theta\rho-\theta+\rho-2)+(\theta-1)(\rho-2)+3n(-\zeta(2\theta\rho+\rho-3)+2\theta+\rho-3))}{6(\beta-1)\theta^3} \tag{J.2}$$

Similar expressions, albeit more involved, hold for variance and kurtosis. The main take-away is straightforward: as long as global shocks hit countries with different intensities (or transmit across borders asymmetrically), the gains from risk sharing are not zero, and will generally differ across borders.

We conclude with a comment on the maintained view that the benefits from international insurance are small if shocks are positively correlated across borders or have global nature. From our analysis above, it follows that a higher degree of correlation of GDP across countries may reduce or increase the gains from risk sharing, depending on the underlying heterogeneity of shock intensity across borders. In this sense, the high degree of dispersion in the distribution of moments that we document in the main text is likely to generate gains from insurance dominate the mitigating effect of positive correlations.

K PWT list of countries and moments

We use data from the Penn World Table (PWT) 10.1. Table 2 lists the countries available in the dataset and the related iso3 abbreviations. Table 3 displays empirical moments for a sub-set of these countries.

Table 2: List of countries

ABW = Aruba	FRA = France	MYS = Malaysia
AGO = Angola	GAB = Gabon	NAM = Namibia
AIA = Anguilla	GBR = United Kingdom	NER = Niger
ALB = Albania	GHA = Ghana	NGA = Nigeria
ARE = United Arab Emirates	GIN = Guinea	NIC = Nicaragua
ARG = Argentina	GMB = Gambia	NLD = Netherlands
ATG = Antigua and Barbuda	GNB = Guinea-Bissau	NOR = Norway
AUS = Australia	GNQ = Equatorial Guinea	NPL = Nepal
AUT = Austria	GRC = Greece	NZL = New Zealand
BDI = Burundi	GRD = Grenada	OMN = Oman
BEL = Belgium	GTM = Guatemala	PAK = Pakistan
BEN = Benin	HKG = China, Hong Kong SAR	PAN = Panama
BFA = Burkina Faso	HND = Honduras	PER = Peru
BGD = Bangladesh	HTI = Haiti	PHL = Philippines
BGR = Bulgaria	HUN = Hungary	POL = Poland
BHR = Bahrain	IDN = Indonesia	PRT = Portugal
BHS = Bahamas	IND = India	PRY = Paraguay
BLZ = Belize	IRL = Ireland	PSE = State of Palestine
BMU = Bermuda	IRN = Iran (Islamic Republic of)	QAT = Qatar
BOL = Bolivia (Plurinational State of)	IRQ = Iraq	ROU = Romania
BRA = Brazil	ISL = Iceland	RWA = Rwanda
BRB = Barbados	ISR = Israel	SAU = Saudi Arabia
BRN = Brunei Darussalam	ITA = Italy	SDN = Sudan
BTN = Bhutan	JAM = Jamaica	SEN = Senegal
BWA = Botswana	JOR = Jordan	SGP = Singapore
CAF = Central African Republic	JPN = Japan	SLE = Sierra Leone
CAN = Canada	KEN = Kenya	SLV = El Salvador
CHE = Switzerland	KHM = Cambodia	STP = Sao Tome and Principe
CHL = Chile	KNA = Saint Kitts and Nevis	SUR = Suriname
CHN = China	KOR = Republic of Korea	SWE = Sweden
CIV = Cote d'Ivoire	KWT = Kuwait	SWZ = Eswatini
CMR = Cameroon	LAO = Lao People's DR	SYC = Seychelles
COD = Congo, Democratic Republic	LBN = Lebanon	SYR = Syrian Arab Republic
COG = Congo	LBR = Liberia	TCA = Turks and Caicos Islands
COL = Colombia	LCA = Saint Lucia	TCD = Chad
COM = Comoros	LKA = Sri Lanka	TGO = Togo
CPV = Cabo Verde	LSO = Lesotho	THA = Thailand
CRI = Costa Rica	LUX = Luxembourg	TTO = Trinidad and Tobago
CYM = Cayman Islands	MAC = China, Macao SAR	TUN = Tunisia
CYP = Cyprus	MAR = Morocco	TUR = Turkey
DEU = Germany	MDG = Madagascar	TWN = Taiwan
DJI = Djibouti	MDV = Maldives	TZA = U.R. of Tanzania: Mainland
DMA = Dominica	MEX = Mexico	UGA = Uganda
DNK = Denmark	MLI = Mali	URY = Uruguay
DOM = Dominican Republic	MLT = Malta	USA = United States of America
DZA = Algeria	MMR = Myanmar	VCT = St. Vincent & Grenadines
ECU = Ecuador	MNG = Mongolia	VEN = Venezuela (Bolivarian Republic of)
EGY = Egypt	MOZ = Mozambique	VGB = British Virgin Islands
ESP = Spain	MRT = Mauritania	VNM = Viet Nam
ETH = Ethiopia	MSR = Montserrat	ZAF = South Africa
FIN = Finland	MUS = Mauritius	ZMB = Zambia
FJI = Fiji	MWI = Malawi	ZWE = Zimbabwe

Table 3: Sample Moments

Country	Population	Stdev	Skewness	Kurtosis	Population	Country	Stdev	Skewness	Kurtosis
AGO	31.825	0.047	-2.421	14.824	ISR	8.519	0.030	-0.953	4.646
ARG	44.781	0.043	-0.929	4.291	ITA	60.550	0.018	0.033	4.191
AUS	25.203	0.017	-0.956	5.384	JPN	126.860	0.019	-0.589	3.897
AUT	8.955	0.017	-0.377	3.485	KOR	51.225	0.029	-1.163	4.969
BEL	11.539	0.013	-0.532	3.409	LUX	0.616	0.027	0.236	2.814
BGR	7.000	0.034	-0.381	6.536	MEX	127.576	0.027	-0.626	4.106
BOL	11.513	0.034	-0.916	7.138	MLT	0.440	0.030	0.020	2.198
BRA	211.050	0.027	-0.243	3.025	MYS	31.950	0.029	-0.613	4.716
CAN	37.411	0.018	-0.663	4.144	NAM	2.495	0.023	0.426	2.997
CHE	8.591	0.019	-0.864	4.434	NLD	17.097	0.017	-0.352	2.613
CHL	18.952	0.042	-0.821	4.852	NOR	5.379	0.012	0.110	2.550
CHN	1433.784	0.045	-1.331	7.011	NZL	4.783	0.027	0.016	2.800
COL	50.339	0.017	-0.447	3.181	PER	32.510	0.037	-0.141	4.846
CYP	0.868	0.049	-1.787	8.975	PHL	108.117	0.025	-1.297	6.592
CZE	10.689	0.021	-0.995	9.421	POL	37.888	0.031	-1.526	6.180
DEU	83.517	0.017	-0.889	5.109	PRT	10.226	0.023	-0.504	4.263
DNK	5.772	0.018	-0.280	2.736	PRY	7.045	0.029	0.055	3.013
EGY	100.388	0.030	1.159	5.869	QAT	2.832	0.039	1.089	6.217
ESP	46.737	0.027	-0.686	5.938	ROU	19.365	0.038	-0.464	3.506
EST	1.326	0.038	-2.121	12.134	SAU	34.269	0.058	-0.538	3.464
FIN	5.532	0.025	-0.513	3.046	SGP	5.804	0.030	-0.840	3.593
FRA	67.351	0.012	-0.646	3.795	SVK	5.457	0.027	-1.050	9.593
GBR	67.530	0.017	0.131	3.374	SVN	2.079	0.020	-1.196	8.220
GRC	10.473	0.030	-0.679	3.602	SWE	10.036	0.016	-0.969	4.786
HKG	7.436	0.032	-0.237	2.888	THA	69.626	0.041	0.730	9.189
HUN	9.685	0.023	-1.752	8.569	TUN	11.695	0.030	0.975	8.120
IDN	270.626	0.026	-1.493	10.713	TUR	83.430	0.042	-0.286	3.306
IND	1366.418	0.022	-0.168	4.296	TWN	23.596	0.022	-0.220	2.672
IRL	4.882	0.031	1.108	8.151	URY	3.462	0.032	-0.091	2.758
IRN	82.914	0.084	-1.840	11.391	USA	329.065	0.017	-0.601	3.304
ISL	0.339	0.035	-0.450	2.802	ZAF	58.558	0.016	0.045	2.862

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